Game Theory Lecture #6

Outline:

- Hawk/Dove & Mixed strategies
- Correlated equilibrium

Hawk/Dove

	H	D
H	$0,0$	$6,1$
D	$1,6$	$3,3$

- Setup:
  - H: hawk = aggressive
  - D: dove = passive
  - Model of game of "chicken" or traffic intersection
- First look: What are the pure (i.e., non-randomized) action NE?
  - Best response function for row player:

$$B_{\text{ROW}}(H) = D \;\;\&\;\; B_{\text{ROW}}(D) = H$$

  - Symmetric for column player
  - NE: (H, D) and (D, H)

Hawk/Dove: Mixed strategies

	H	D
H	$0,0$	$6,1$
D	$1,6$	$3,3$

- Second look: What are the mixed strategy NE?
- As before, we construct best response function, but for *mixed* strategies
  - Row: Pr $(H) = p$ and Pr $(D) = 1 - p$
  - Column: Pr $(H) = q$ and Pr $(D) = 1 - q$
  - Players select $\{H, D\}$ *independently*
- Best response for row player: Need to maximize expected payoff, i.e.,

$$\max_{0 \le p \le 1} p\Big(0 \cdot q + 6 \cdot (1-q)\Big) + (1-p)\Big(1 \cdot q + 3 \cdot (1-q)\Big)$$

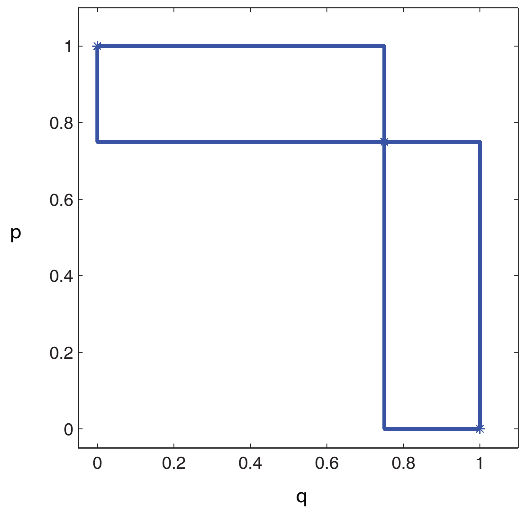
$$\Downarrow$$

$$B_{\text{ROW}}(q) = \begin{cases} 1 & \Big(0 \cdot q + 6 \cdot (1-q)\Big) > \Big(1 \cdot q + 3 \cdot (1-q)\Big) \\ [0,1] & \Big(0 \cdot q + 6 \cdot (1-q)\Big) = \Big(1 \cdot q + 3 \cdot (1-q)\Big) \\ 0 & \Big(0 \cdot q + 6 \cdot (1-q)\Big) < \Big(1 \cdot q + 3 \cdot (1-q)\Big) \end{cases}$$

- Conclusion:

$$B_{\text{ROW}}(q) = \begin{cases} 1 & q < 3/4 \\ [0,1] & q = 3/4 \\ 0 & q > 3/4 \end{cases} \ \& \ B_{\text{COL}}(p) = \begin{cases} 1 & p < 3/4 \\ [0,1] & p = 3/4 \\ 0 & p > 3/4 \end{cases}$$

H/D: Best response plots



- NE occur at intersection of best response plots
  - NE of original pure strategy game are still present
  - New "mixed strategy" NE: $(p^*, q^*) = (3/4, 3/4)$
- Peculiarity: At mixed strategy NE, players are *indifferent*, i.e.,

$$B_{\text{ROW}}(3/4) = [0, 1] \ \ \& \ \ B_{\text{COL}}(3/4) = [0, 1]$$

i.e., at NE, best response is to play (H, D) with any probability combination.

- The mixed strategy NE makes *both players* indifferent

H/D: Coordination

	H	D
H	$0, 0$	$6, 1$
D	$1, 6$	$3, 3$

- Think of Hawk/Dove game as model of traffic intersection
- NE:
  - Row player always has right of way
  - Column player always has right of way
  - Both player proceeds with probability $3/4$

  None of these are very satisfying
- Alternative "solution": Players alternate right of way
- Compare payoffs (row player):
  - At mixed strategy NE:

    $$0 \cdot pq + 6 \cdot p(1-q) + 1 \cdot (1-p)q + 3 \cdot (1-p)(1-q) = 1.5$$

    at $p = q = 3/4$
  - Alternating:

    $$6 \cdot 1/2 + 1 \cdot 1/2 = 3.5$$
- Alternating is desirable...can it be supported from game theoretic viewpoint?

H/D correlated (non-independent) strategies

- Alternating can be modeled as randomly choose who gets right of way. This results in probability distribution of the form:

0	1/2
1/2	0

- Mixed strategy forces probability distributions over joint actions with special form:

pq	$p(1-q)$
$(1-p)q$	$(1-p)(1-q)$

  Mixed strategies *cannot* produce above probabilities

- New setup:
  - Players, actions, preferences over action profile probabilities (as before)
  - *Referee* will choose an action profile according to a "joint distribution"

α_{HH}	α_{HD}
α_{DH}	α_{DD}

  - Player decisions: Obey referee or not.

- A **correlated equilibrium** is a Referee's probability distribution, α^*, such that neither player has a unilateral incentive to disobey.

H/D correlated equilibrium

- Claim:

0	$1/2$
$1/2$	0

  is a correlated equilibrium

- Analyze from row player's perspective: What is expected reward?

	H	D
H	$0, 0$	$6, 1$
D	$1, 6$	$3, 3$

  – Obey referee: $(1 + 6)/2 = 3.5$
  – Obey referee for H but deviate when referee says D: $(0 + 6)/2 = 3$
  – Obey referee for D but deviate when referee says H: $(1 + 3)/2 = 2$
  – Always disobey referee: $(0 + 3)/2 = 1.5$

  Conclusion: There is no *unilateral* incentive to disobey referee.

- Correlated equilibrium include:
  – Pure strategy NE
  – Mixed strategy NE
  – Distributions that are neither Pure or Mixed NE
  – Any convex combination of all of the above

- Models of correlation:
  – Referee draws a^* ⇒ Information given to each player ⇒ Players can adjust strategy
  – Correlated equilibrium: Information given to each player is a_i^*. (Conditional)
  – Coarse correlated equilibrium: No information given to each player. (Unconditional)
  – Which equilibrium set is larger? Does one contain the other?

H/D correlated equilibrium

- Can we characterize the set of coarse correlated equilibria?

	H	D
H	$0, 0$	$6, 1$
D	$1, 6$	$3, 3$

- Row player's perspective:
  - Expected payoff obeying referee:
  $$U_{\mathrm{ROW}}(\alpha^*) = \alpha^*_{\mathrm{HH}} \cdot 0 + \alpha^*_{\mathrm{HD}} \cdot 6 + \alpha^*_{\mathrm{DH}} \cdot 1 + \alpha^*_{\mathrm{DD}} \cdot 3$$
  - Expected payoff by choosing H:
  $$U_{\mathrm{ROW}}(\alpha^*|H) = (\alpha^*_{\mathrm{HH}} + \alpha^*_{\mathrm{DH}}) \cdot 0 + (\alpha^*_{\mathrm{HD}} + \alpha^*_{\mathrm{DD}}) \cdot 6$$
  - Expected payoff by choosing D:
  $$U_{\mathrm{ROW}}(\alpha^*|D) = (\alpha^*_{\mathrm{HH}} + \alpha^*_{\mathrm{DH}}) \cdot 1 + (\alpha^*_{\mathrm{HD}} + \alpha^*_{\mathrm{DD}}) \cdot 3$$
- Coarse correlated equilibrium conditions:

$$\begin{array}{rcl}
U_{\mathrm{ROW}}(\alpha^*) & \geq & U_{\mathrm{ROW}}(\alpha^*|H) \;\Rightarrow\; \alpha^*_{\mathrm{DH}} \geq 3\alpha^*_{\mathrm{DD}} \\
U_{\mathrm{ROW}}(\alpha^*) & \geq & U_{\mathrm{ROW}}(\alpha^*|D) \;\Rightarrow\; \alpha^*_{\mathrm{HD}} \geq 3\alpha^*_{\mathrm{HH}} \\
U_{\mathrm{COL}}(\alpha^*) & \geq & U_{\mathrm{COL}}(\alpha^*|H) \;\Rightarrow\; \alpha^*_{\mathrm{HD}} \geq 3\alpha^*_{\mathrm{DD}} \\
U_{\mathrm{COL}}(\alpha^*) & \geq & U_{\mathrm{COL}}(\alpha^*|D) \;\Rightarrow\; \alpha^*_{\mathrm{DH}} \geq 3\alpha^*_{\mathrm{HH}}
\end{array}$$

- Set of coarse correlated equilibrium:

α^*_{HH}	$\alpha^*_{\mathrm{HD}} \geq \max\{1/3 \cdot \alpha^*_{\mathrm{HH}}, 3 \cdot \alpha^*_{\mathrm{DD}}\}$
$\alpha^*_{\mathrm{DH}} \geq \max\{1/3 \cdot \alpha^*_{\mathrm{HH}}, 3 \cdot \alpha^*_{\mathrm{DD}}\}$	α^*_{DD}

- Is $(\alpha^*_{\mathrm{HH}}, \alpha^*_{\mathrm{HD}}, \alpha^*_{\mathrm{DH}}, \alpha^*_{\mathrm{DD}}\} = \{0, 1/2, 1/2, 0\}$ a coarse correlated equilibrium?
- What is the maximum value of α^*_{HH} in any coarse correlated equilibrium?