Lecture 8:
Stackelberg game

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- Example of sequential game with continuous strategy space
- Power of backward induction to find the equilibrium
- Example: Stackelberg competition
  - Sequential version of Cournot dupoloy
- Stackelberg game
  - One player (the "leader") moves first, and all other players (the "followers") move after him.

Competition between two firms: Model

- Two firms ($N = 2$)
- Each firm chooses a quantity $s_n \geq 0$
- Cost of producing s_n : $c_n s_n$

- *Demand (or Pricing) curve*:

  Price = $P(s_1 + s_2) = a - b(s_1 + s_2)$

- Payoffs:

  Profit = $\Pi_n(s_1, s_2) = P(s_1 + s_2)\, s_n - c_n s_n$

Simultaneous Play: Cournot Competition
(We've covered this earlier)

Best response

- *Assume* $c_1 = c_2 = c$

- *Best response set* for player *n* to s_{-n}:

$$R_n(\mathbf{s}_{-n}) = \arg\max_{s_n \in S_n} \Pi_n(s_n, \mathbf{s}_{-n})$$

- Note: $\arg\max_{x \in X} f(x)$ is the *set* of x that maximize $f(x)$

Example: Cournot duopoly

- Calculating the best response given s_{-n}:

$$\max_{s_n \geq 0} [(a - bs_n - bs_{-n})s_n - cs_n] \implies$$

- Differentiate and solve:

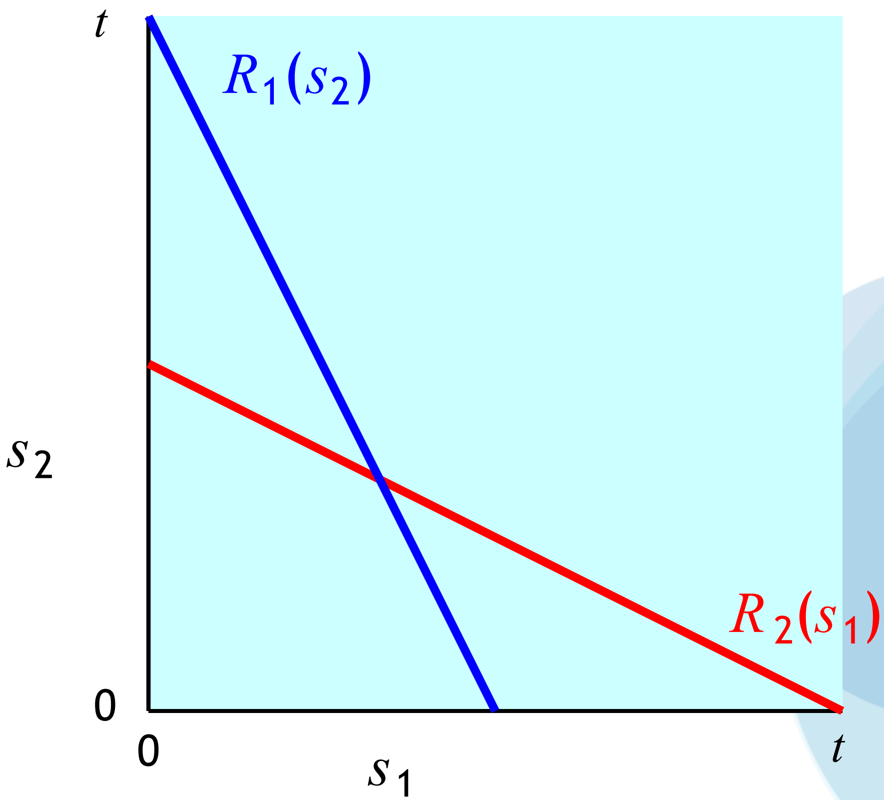
$$a - c - bs_{-n} - 2bs_n = 0$$

- So the best response function is:

$$R_n(s_{-n}) = \left[\frac{a-c}{2b} - \frac{s_{-n}}{2}\right]^+$$

Example: Cournot duopoly

- For simplicity, let $t = (a - c)/b$



Sequential Play:
Stackelberg Competition

Model Again

- Two firms ($N = 2$)
- Each firm chooses a quantity $s_n \geq 0$
- Cost of producing s_n : $c_n s_n$

- *Demand (or Pricing) curve*:

  Price = $P(s_1 + s_2) = a - b(s_1 + s_2)$

- Payoffs:

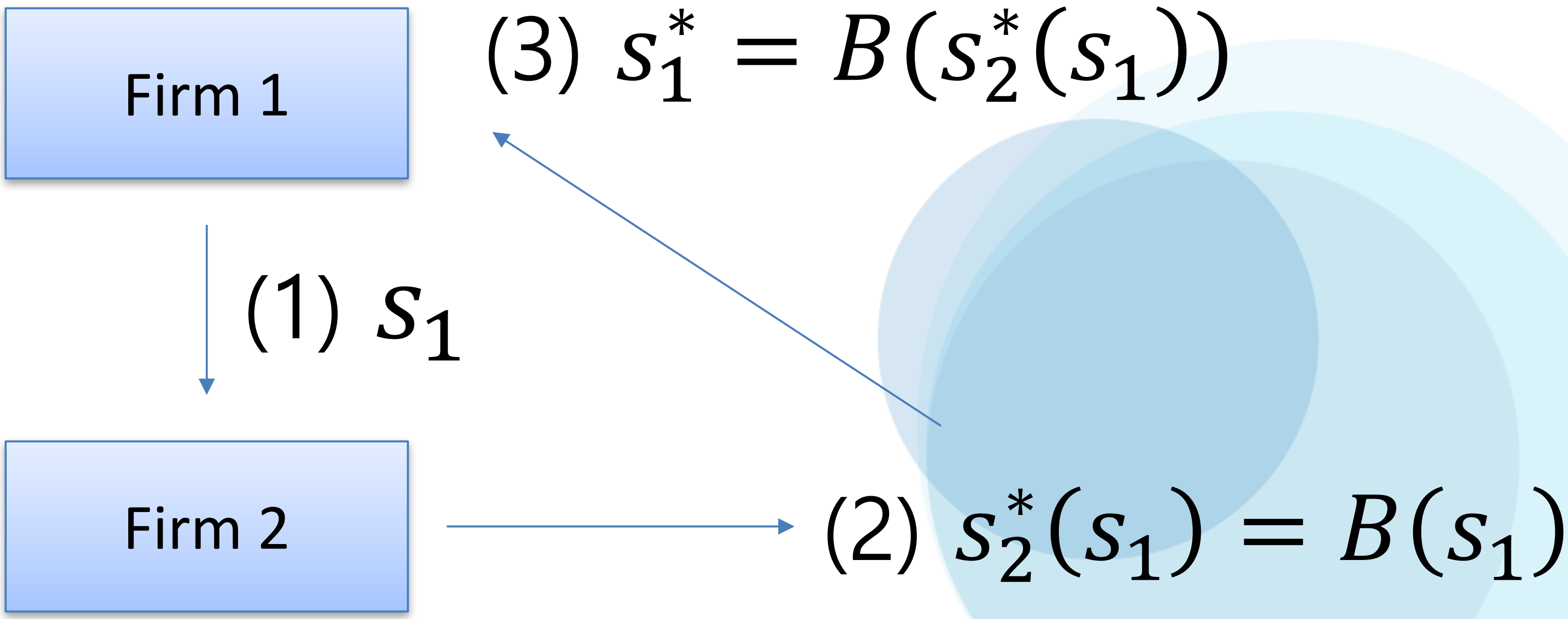
  Profit = $\Pi_n(s_1, s_2) = P(s_1 + s_2)\, s_n - c_n s_n$

Stackelberg Competition

- Firm 1 moves before firm 2.
- Firm 2 observes firm 1's quantity choice s_1, then chooses s_2.
- Interesting question
  - How does the equilibrium change in this case?
  - Advantageous for firm 1 or firm 2?

Finding the NE: Backward Induction

- We solve the game using *backward induction*.



Firm 2: Second Stage

- We solve the game using *backward induction*.
- Start with second stage:

  Given s_1, firm 2 chooses s_2 as

  $s_2 = \arg\max_{s_2 \in S_2} \Pi_2(s_1, s_2)$

- This is the *best response* $R_2(s_1)$!

Best response for firm 2

- Recall the best response given s_1:

$$\max_{s_2 \geq 0} [(a - bs_2 - bs_1)s_2 - c_2 s_2] \implies$$

- Differentiate and solve:

$$a - c_2 - bs_1 - 2bs_2 = 0$$

- So:

$$R_2(s_1) = \left[\frac{a - c_2}{2b} - \frac{s_1}{2}\right]^+$$

Firm 1's decision

- Backward induction:
- Maximize firm 1's decision, *accounting for firm 2's response at stage 2.*
- Thus firm 1 chooses s_1 as

$$s_1 = \arg\max_{s_1 \in S_1} \Pi_1(s_1, R_2(s_1))$$

Firm 1's decision

- Define $t_n = (a - c_n)/b$.
- If $s_1 \leq t_2$, then payoff to firm 1 is:

$$\Pi_1 = \left(a - bs_1 - b\left(\frac{t_2}{2} - \frac{s_1}{2}\right)\right) s_1 - c_1 s_1$$

- If $s_1 > t_2$, then payoff to firm 1 is:

$$\Pi_1 = (a - bs_1)\, s_1 - c_1 s_1$$

Firm 1's decision

- For simplicity, we assume that $2c_2 \leq a + c_1$
- This assumption ensures that

$$(a - bs_1)\, s_1 - c_1 s_1$$

- is *strictly decreasing* for $s_1 > t_2$.

- Thus firm 1's optimal s_1 must lie in $[0, t_2]$.

Firm 1's decision

- If $s_1 \leq t_2$, then payoff to firm 1 is:

$$\Pi_1 = \left(a - bs_1 - b\left(\frac{t_2}{2} - \frac{s_1}{2}\right)\right) s_1 - c_1 s_1$$

Firm 1's decision

- If $s_1 \leq t_2$, then payoff to firm 1 is:

$$\Pi_1 = \left(\frac{a}{2} - \frac{b}{2}s_1 + \frac{c_2}{2}\right) s_1 - c_1 s_1$$

Firm 1's decision

- If $s_1 \leq t_2$, then payoff to firm 1 is:

$$\Pi_1 = -\frac{b}{2}s_1^2 + \left(\frac{a}{2} + \frac{c_2}{2} - c_1\right)s_1$$

- Thus optimal s_1 is:

$$s_1 = \frac{a - 2c_1 + c_2}{2b}$$

Stackelberg equilibrium

- So what is the Stackelberg equilibrium?
- *Must give complete strategies:*

  $s_1^* = (a - 2c_1 + c_2)/2b$

  $s_2^*(s_1) = (t_2/2 - s_1/2)^+$

- The *equilibrium outcome* is that firm 1 plays s_1^*, and firm 2 plays $s_2^*(s_1^*)$.

Comparison:
Simultaneous Play vs. Sequential Play

Comparison to Cournot

- Assume $c_1 = c_2 = c$.

- In Cournot equilibrium:
  (1) $s_1 = s_2 = t/3$.
  (2) $\Pi_1 = \Pi_2 = (a - c)^2/(9b)$.

- In Stackelberg equilibrium:
  (1) $s_1 = t/2$, $s_2 = t/4$.
  (2) $\Pi_1 = (a - c)^2/(8b)$, $\Pi_2 = (a - c)^2/(16b)$

Comparison to Cournot

- So in Stackelberg competition:
- The *leader* has *higher* profits
- The *follower* has *lower* profits
- This is called a *first mover advantage.*

Stackelberg competition: moral

- *Moral:*

  Additional information available can lower a player's payoff, if it is common knowledge that the player will have the additional information.

  (*Here:* firm 1 takes advantage of knowing firm 2 knows s_1.)

Summary