Lecture 3: Normal-form game (Strategic-form game) with mixed strategies

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Key Words

- Normal-form (Strategic form) Game
- Matrix game
  - Strategy spaces are discrete
- Continuous-kernel game
  - Strategy spaces are continuous

- Strictly dominated strategies
- Pure/Mixed strategy
- Saddle point, Nash equilibrium

Matrix Game: Mixed Strategy

Pure vs. Mixed Strategies

- Previous lecture
  - Players make **deterministic choices** from their strategy spaces

- Strategies are **pure** if a player *i* selects, in a deterministic manner (probability 1), one strategy out of its strategy set S_i

- Players can also select a **probability distribution** over their set of strategies, in which cases the strategies are called **mixed**

Mixed Strategies

- Assume a zero-sum game, again.
- Payoffs are computed as expectations

		1/3	2/3
		C	D
Player 1	A	4,-4	0,0
	B	-5,5	3,-3

Payoff to P1 when playing A = 1/3(4) + 2/3(0) = 4/3
Payoff to P1 when playing B = 1/3(-5) + 2/3(3) = 1/3

- How should players choose prob. distribution?

Mixed Strategies

- No pure strategy NE
- Claim 1
  - There exists a stochastic equilibrium with both playing (1/2,1/2) strategy

		P2	
		Head	Tail
P1	Head	1,-1	-1,1
	Tail	-1,1	1,-1

- Why?
  - Need to argue that if P2 chooses (1/2,1/2) strategy, P1 optimally chooses (1/2,1/2) strategy, and vice versa
  - Suppose that P2(1/2,1/2) and P1(p,1-p)
    - Each outcome (H,H) and (H,T) occurs w.p. p/2.
    - Each outcome (H,T) and (T,H) occurs w.p. (1-p)2.
  - P1 gains 1 w.p. p/2 + (1-p)/2 = ½, and loses 1 w.p. ½.
  - P1's payoff does not depend on p!. Thus, every p (including 1/2) is optimal.
  - Similarly, for P2.

Mixed Nash Equilibrium

- Define σ_i as a probability mass function over S_i, the set of actions of player *i*
- When working with mixed strategies, each player *i* aim to maximize their **expected payoff**

$$u_i(\sigma) = \sum_{s \in S} \left(\prod_{j=1}^{I} \sigma_j(s_j) \right) u_i(s)$$

- Mixed strategies Nash equilibrium

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*) \qquad \text{for all } \sigma_i \in \Sigma_i.$$

Mixed Nash Equilibrium

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Aside: Best Response Function

- Assume only pure strategy

Definition 9 *The* best response function $b_i(\mathbf{s}_{-i})$ *of a player* i *to the profile of strategies* $\mathbf{s}_{-i}$ *is a set of strategies for that player such that*

$$b_i(\mathbf{s}_{-i}) = \{s_i \in \mathcal{S}_i \mid u_i(s_i, \mathbf{s}_{-i}) \geq u_i(s_i', \mathbf{s}_{-i}),\ \forall s_i' \in \mathcal{S}_i\}. \qquad (3.5)$$

- Best response function is a set-valued function
- Probably, we can define NE based on Best Response?

Aside: BR and NE

Proposition 1 *A strategy profile* $\mathbf{s}^* \in \mathcal{S}$ *is a* Nash equilibrium *of a noncooperative game if and only if every player's strategy is a* best response *to the other players' strategies, that is:*

$$s_i^* \in b_i(\mathbf{s}_{-i}^*) \text{ for every player } i. \qquad (3.6)$$

- BR and NE can be similarly defined for mixed strategies

Computing NE using the BR concept

		P2	
		Head	Tail
P1	Head	1,-1	-1,1
	Tail	-1,1	1,-1

- P1(p,1-p), P2(q,1-q)

- Payoff to P1 when playing Head = q + (1-q)(-1) = 2q -1
- Payoff to P1 when playing Tail = q(-1) + (1-q)1 = 1- 2q

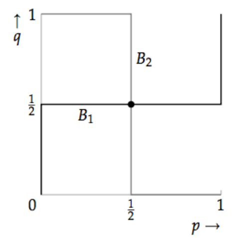
- If q < ½, P1's payoff (Tail) > P1's payoff (Head)
  → P1's payoff (1,0) > P1's payoff (p, 1-p) for p > 0.
- Similarly, if q > ½, P1's payoff (0,1) > P1's payoff (p, 1-p) for p > 0.
- If q = ½, P1's payoff (1,0) = P1's payoff (0,1) = P1's payff (p,1-p).
- Thus, P1's best response to P2(q,1-q) is:

- Thus, P1's best response to P2(q,1-q) is:

$$B_1(q) = \begin{cases} \{0\} & \text{if } q < \frac{1}{2} \\ \{p : 0 \leq p \leq 1\} & \text{if } q = \frac{1}{2} \\ \{1\} & \text{if } q > \frac{1}{2}. \end{cases}$$

- Similarly, P2's best response to P1(p,1-p) is:

The best response function of player 2 is similar: $B_2(p) = \{1\}$ if $p < \frac{1}{2}$, $B_2(p) = \{q : 0 \leq q \leq 1\}$ if $p = \frac{1}{2}$, and $B_2(p) = \{0\}$ if $p > \frac{1}{2}$. Both best response functions are illustrated in Figure 110.1.



Exercise: Battle of Sexes

- What is the mixed strategy NE?

		Husband	
		LW	WL
Wife	LW	2, 1	0, 0
	WL	0, 0	1, 2

Summary