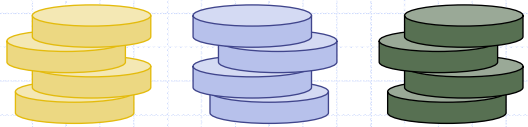
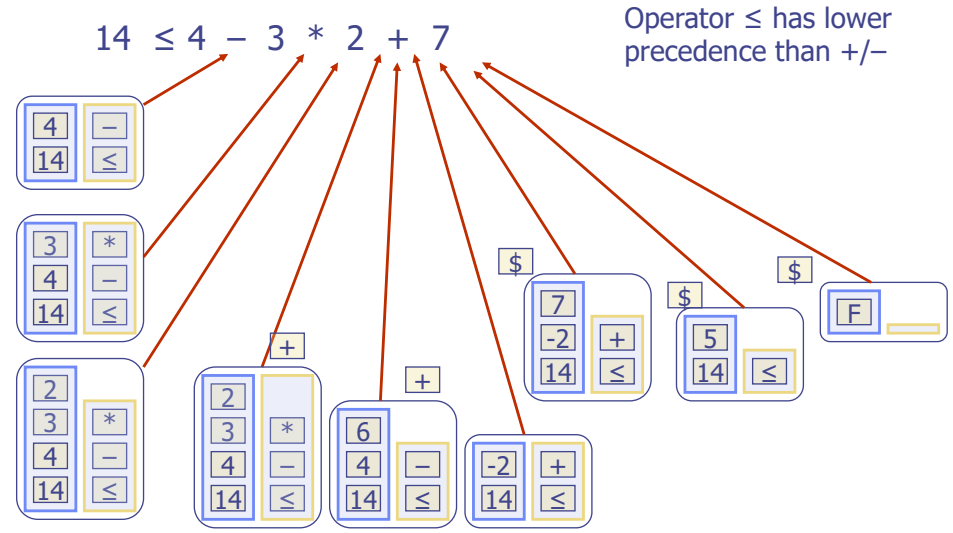
Stacks

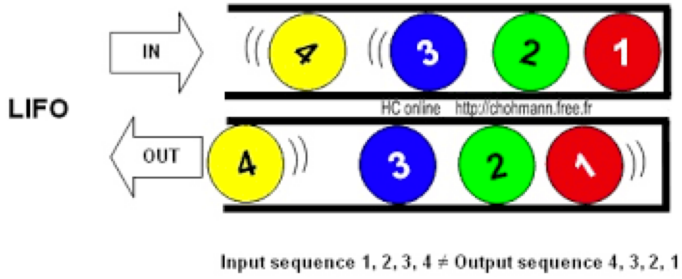


Example: Algorithm on an Example Expression



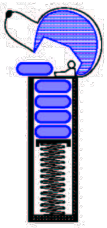
Overview and Reading

- Reading: Chapter 5.1
- Last-In-First-Out Data Structure



The Stack ADT

- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean empty(): indicates whether no elements are stored



Stack Interface in C++

- C++ interface corresponding to our Stack ADT
- Uses an exception class StackEmpty
- Different from the built-in C++ STL class stack
- STL: Standard Template Library

```
template <typename E>
class Stack  {
public:
  int size() const;
  bool empty() const;
  const E& top() const
      throw(StackEmpty);
  void push(const E& e);
  void pop() throw(StackEmpty);
}
```

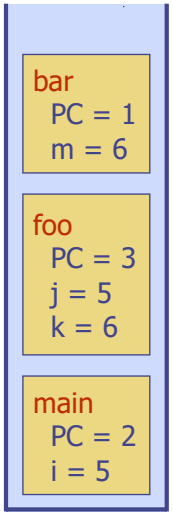

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the C++ run-time system
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Example: C++ Run-Time Stack

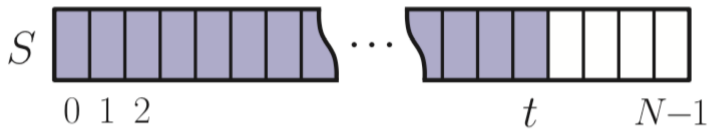
- The C++ run-time system keeps track of the chain of active functions with a stack
- When a function is called, the system pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When the function ends, its frame is popped from the stack and control is passed to the function on top of the stack
- Allows for recursion
- PC: Program Counter

```
main() {
  int i = 5;
  foo(i);
  }

foo(int j) {
  int k;
  k = j+1;
  bar(k);
  }

bar(int m) {
  …
  }
```



Example Implementation: Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element



Example Implementation: Array-based Stack

- A simple way of implementing the Stack ADT
- Add elements from left to right
- A variable keeps track of the index of the top element
- The array storing the stack elements may become full
  - A push operation will then throw a StackFull exception
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

Algorithm size():
  return $t + 1$
Algorithm empty():
  return $(t < 0)$
Algorithm top():
  if empty() **then**
    throw StackEmpty exception
  return $S[t]$
Algorithm push(e):
  if size() $= N$ **then**
    throw StackFull exception
  $t \leftarrow t + 1$
  $S[t] \leftarrow e$
Algorithm pop():
  if empty() **then**
    throw StackEmpty exception
  $t \leftarrow t - 1$

Performance and Limitations

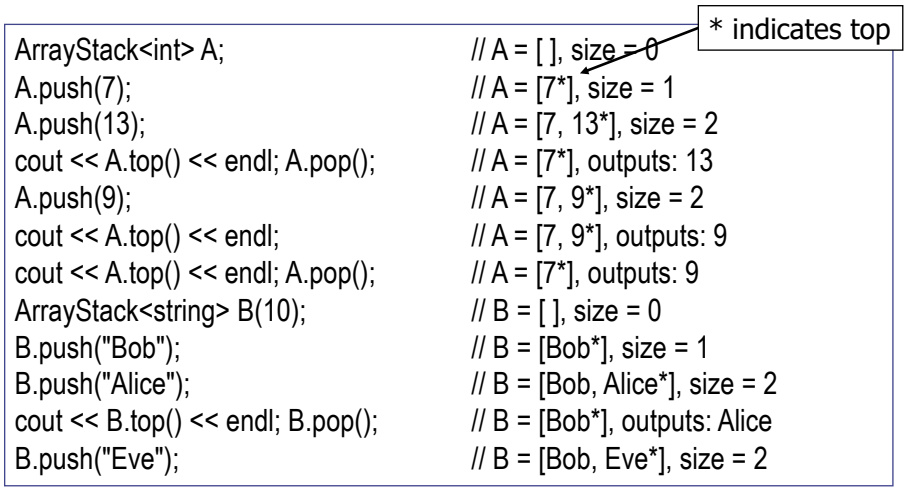
- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception
- Linked-list based Stack in the text (Chapter 5.1.5)

Array-based Stack in C++

```
template <typename E>
class ArrayStack {
private:
   E* S; // array holding the stack
   int cap; // capacity
   int t; // index of top element
public:
   // constructor given capacity
   ArrayStack(int c) :
      S(new E[c]), cap(c), t(-1) { }
```

```
   void pop() {
     if (empty()) throw StackEmpty
           ("Pop from empty stack");
     t--;
    }
   void push(const E& e) {
     if (size() == cap) throw
        StackFull("Push to full stack");
     S[++ t] = e;
}
… (other methods of Stack interface)
```


Example use in C++



Stack in C++ STL

```
#include <stack>
using std::stack;                    // make stack accessible
stack<int> myStack;                  // a stack of integers
```

size(): Return the number of elements in the stack.
empty(): Return true if the stack is empty and false otherwise.
push(e): Push e onto the top of the stack.
pop(): Pop the element at the top of the stack.
top(): Return a reference to the element at the top of the stack.

Example: Parentheses Matching

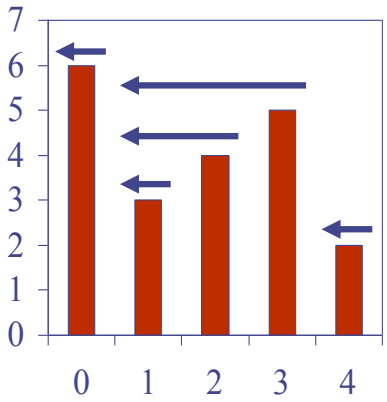
- Each "(", "{", or "[" must be paired with a matching ")", "}", or "["
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

- Good Programmer
  - Someone who thinks that stack is a good data structure for the above task

Example: Computing Spans

- Given an an array X, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \leq X[i]$

- Spans have applications to financial analysis
  - E.g., stock at 52-week high



X	6	3	4	5	2
S	1	1	2	3	1

Algorithm: span1

	i				
X	6	3	4	5	2
S	1	1	2	3	1

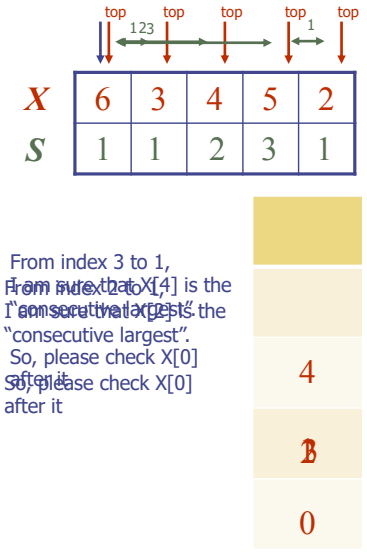
- Loop over i = 0, 1, 2, 3, 4
- For each i, compute S[i]. How?
  - From X[i] downward on, compute the number of elements which are consecutively smaller than X[i]

Quadratic Algorithm

```
Algorithm spans1(X, n)                          #
    Input array X of n integers
    Output array S of spans of X                #
    S ← new array of n integers                 n
    for i ← 0 to n − 1 do                       n
        s ← 1                                   n
        while s ≤ i ∧ X[i − s] ≤ X[i]           1 + 2 + …+ (n − 1)
            s ← s + 1                           1 + 2 + …+ (n − 1)
        S[i] ← s                                n
    return S                                    1
```

- Algorithm *spans1* runs in $O(n^2)$ time

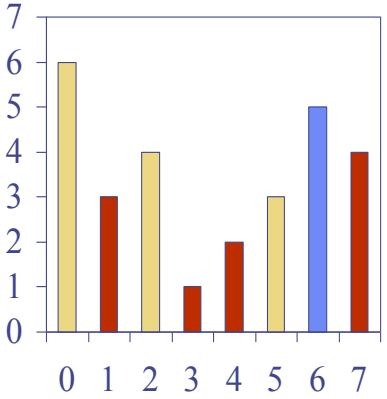
Algorithm: span2




```
Algorithm spans2(X, n)
    S ← new array of n integers
    A ← new empty stack
    for i ← 0 to n − 1 do
        while (¬A.empty() ∧
                X[A.top()] ≤ X[i] ) do
            A.pop()
        if A.empty() then
            S[i] ← i + 1
        else
            S[i] ← i − A.top()
        A.push(i)
    return S
```


Computing Spans with a Stack

- We keep in a stack the indices of the elements visible when "looking back"
- We scan the array from left to right
  - Let i be the current index
  - We pop indices from the stack until we find index j such that $X[i] < X[j]$
  - We set $S[i] \leftarrow i - j$
  - We push x onto the stack



Linear Algorithm

- Each index of the array
  - Is pushed into the stack exactly one
  - Is popped from the stack at most once
- The statements in the while-loop are executed at most n times
- Algorithm ***spans2*** runs in $O(n)$ time

```
Algorithm spans2(X, n)                          #
    S ← new array of n integers                 n
    A ← new empty stack                         1
    for i ← 0 to n − 1 do                       n
        while (¬A.empty() ∧
                X[A.top()] ≤ X[i] ) do          n
            A.pop()                             n
        if A.empty() then                       n
            S[i] ← i + 1                        n
        else
            S[i] ← i − A.top()                  n
        A.push(i)                               n
    return S                                    1
```

Questions?