

# Tips for a good system engineer and/or a good programmer

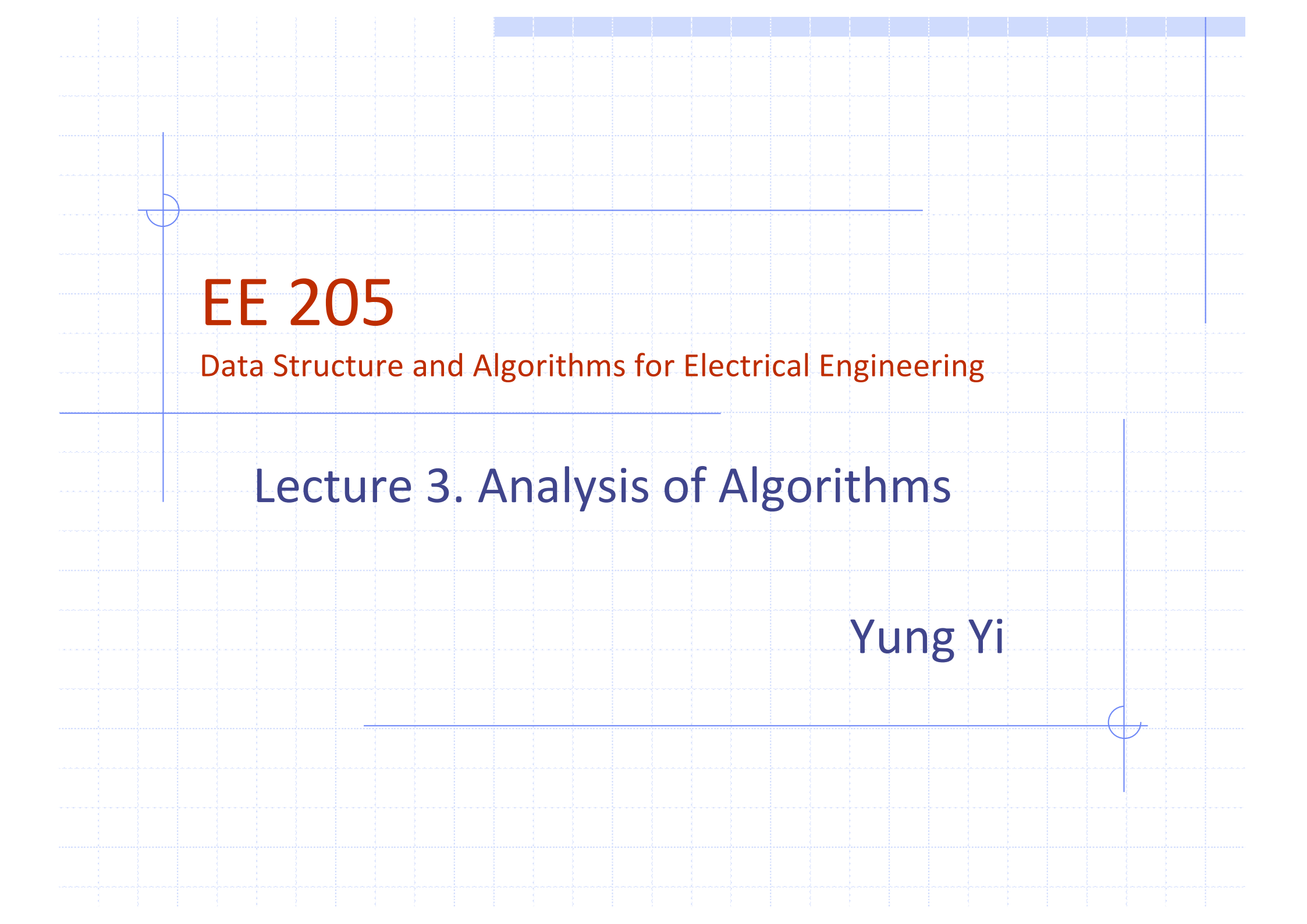
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## ◆ Computer systems

- Whatever you want to do in your computer, there are ways
  - ◆ Fast searching of how to do them in google, and courage to try them in your systems
  - ◆ People often tend to try only what they know
- No fear about using new tools and commands

## ◆ Programming

- Not a technique, but a science (감으로 하는 것이 아님)
- Clearly know what a language provides and understand the underlying principles in relation to its interaction with computer internals

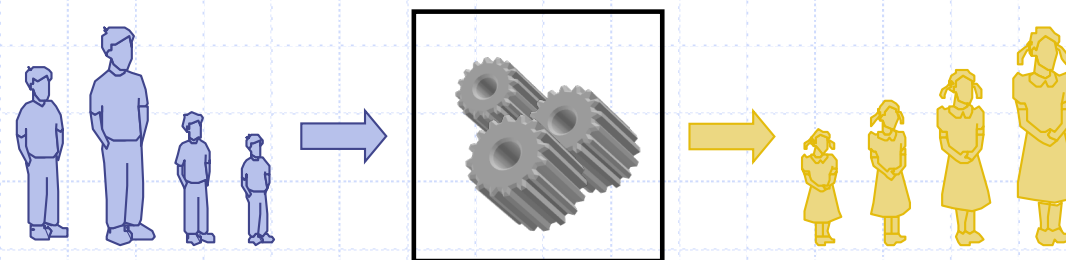


**EE 205**

Data Structure and Algorithms for Electrical Engineering

## Lecture 3. Analysis of Algorithms

Yung Yi



**Input**

**Algorithm**

**Output**

An **algorithm** is a step-by-step procedure for solving a problem in a finite amount of time.

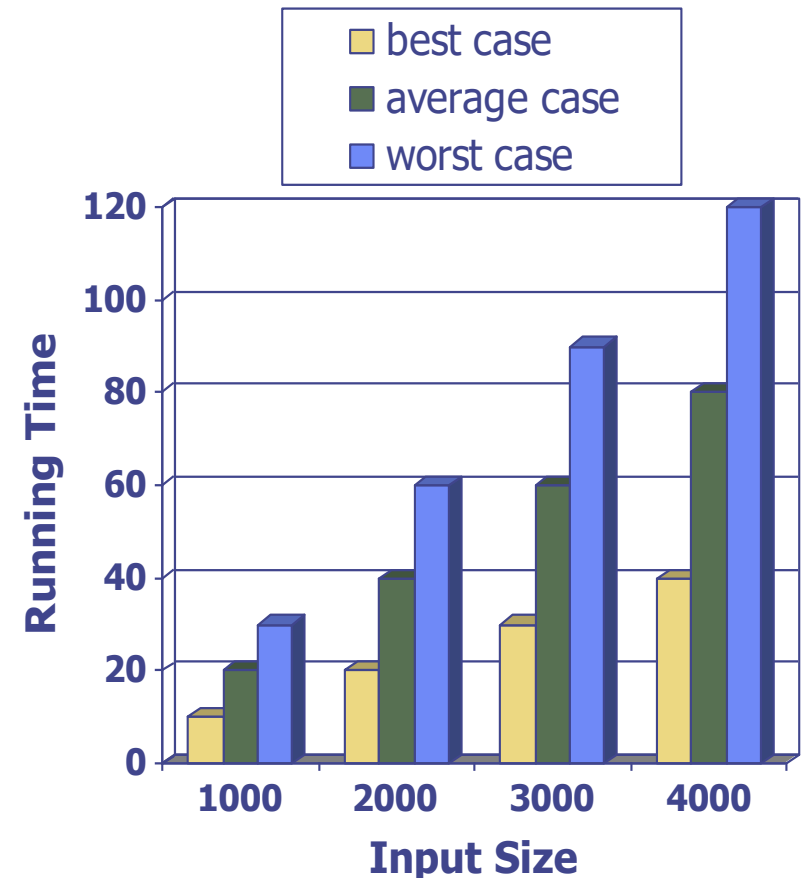
# What are we going to learn?

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- ◆ Need to say that some algorithms are “better” than others
- ◆ Criteria for evaluation
  - Structure of programs (simplicity, elegance, OO, etc.)
  - Running time
  - Memory space
  - What else???

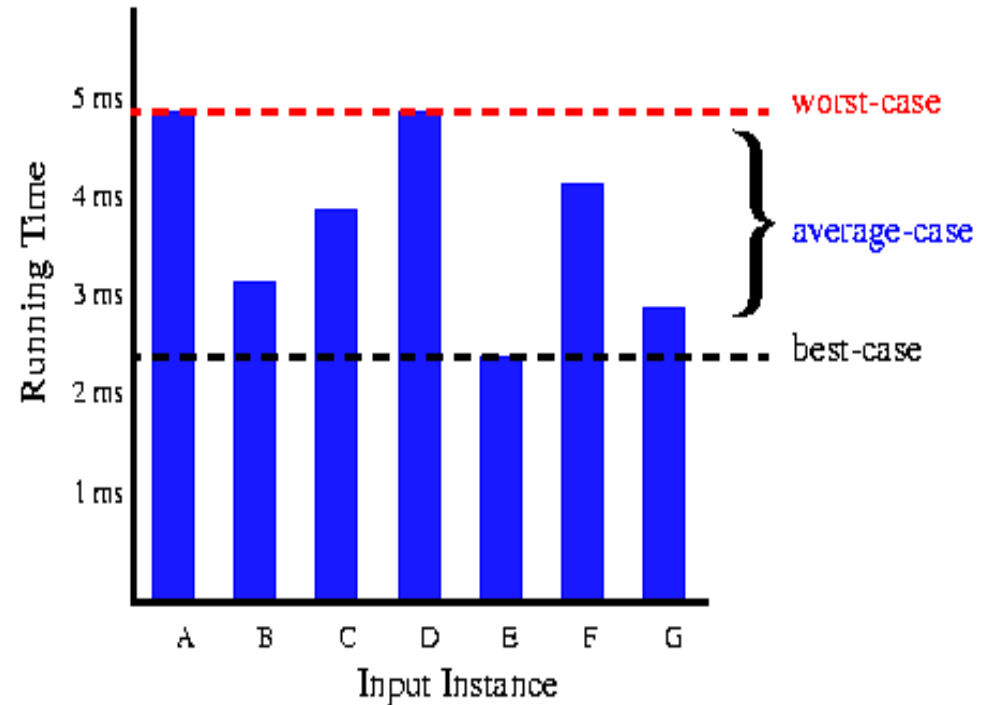
# Running Time (§3.1)

- ◆ Most algorithms transform input objects into output objects.
- ◆ The **running time** of an algorithm typically grows with the input size.
- ◆ **Average-case running time** is often difficult to determine.
  - Why?
- ◆ We focus on the **worst case running time**.
  - Easier to analyze
  - Crucial to applications such as games, finance and robotics



# Average Case vs. Worst Case

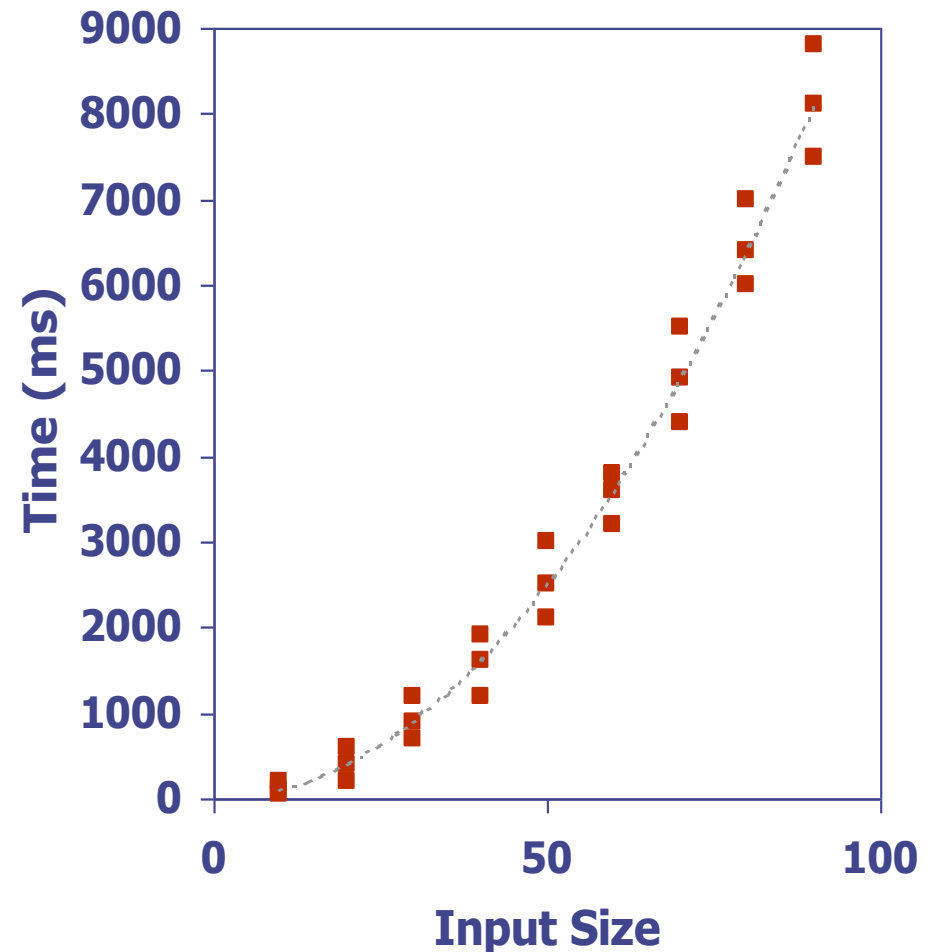
- ◆ The average case running time is harder to analyze because you need to know the probability distribution of the input.
- ◆ In certain apps (air traffic control, weapon systems, etc.), knowing the worst case time is important.



# Experimental Approach

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- ◆ Write a program implementing the algorithm
- ◆ Run the program with inputs of varying size and composition
- ◆ Use a wall clock to get an accurate measure of the actual running time
- ◆ Plot the results



# Limitations of Experiments

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- ◆ It is necessary to implement the algorithm, which may be difficult and often time-consuming
- ◆ Results may not be indicative of the running time on other inputs not included in the experiment.
- ◆ In order to compare two algorithms, the same hardware and software environments must be used
  - Restrictions





# Theoretical Analysis

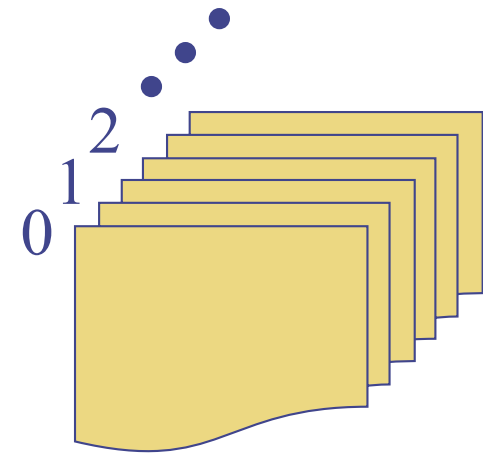
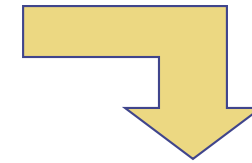
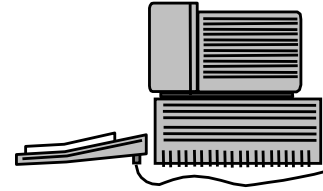
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- ◆ Uses a high-level description of the algorithm instead of an implementation
- ◆ Characterizes running time as a function of the input size,  $n$ .
- ◆ Takes into account all possible inputs
- ◆ Allows us to evaluate the speed of an algorithm *independent of* the hardware/software environment

# The Random Access Machine (RAM) Model

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◆ A CPU



◆ A potentially unbounded bank of **memory** cells, each of which can hold an arbitrary number or character

◆ Memory cells are numbered and accessing any cell in memory takes unit time.

# Pseudocode (§4.2.3)

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- ◆ High-level description of an algorithm
- ◆ More structured than english prose
- ◆ Less detailed than a program
- ◆ Preferred notation for describing algorithms
- ◆ Hides program design issues

Example: find the max element of an array

**Algorithm** *arrayMax*( $A, n$ )

**Input** array  $A$  of  $n$  integers

**Output** maximum element of  $A$

*currentMax*  $\leftarrow A[0]$

**for**  $i \leftarrow 1$  **to**  $n - 1$  **do**

**if**  $A[i] > \textit{currentMax}$  **then**

*currentMax*  $\leftarrow A[i]$

**return** *currentMax*

# Pseudocode Details



## ◆ Control flow

- **if ... then ... [else ...]**
- **while ... do ...**
- **repeat ... until ...**
- **for ... do ...**
- Indentation replaces braces
- 

## ◆ Method declaration

**Algorithm** *method* (*arg* [, *arg...*])

**Input** ...

**Output** ...

## ◆ Method call

*var.method* (*arg* [, *arg...*])

## ◆ Return value

**return** *expression*

## ◆ Expressions

← Assignment  
(like = in C, C++)

= Equality testing  
(like == in C, C++)

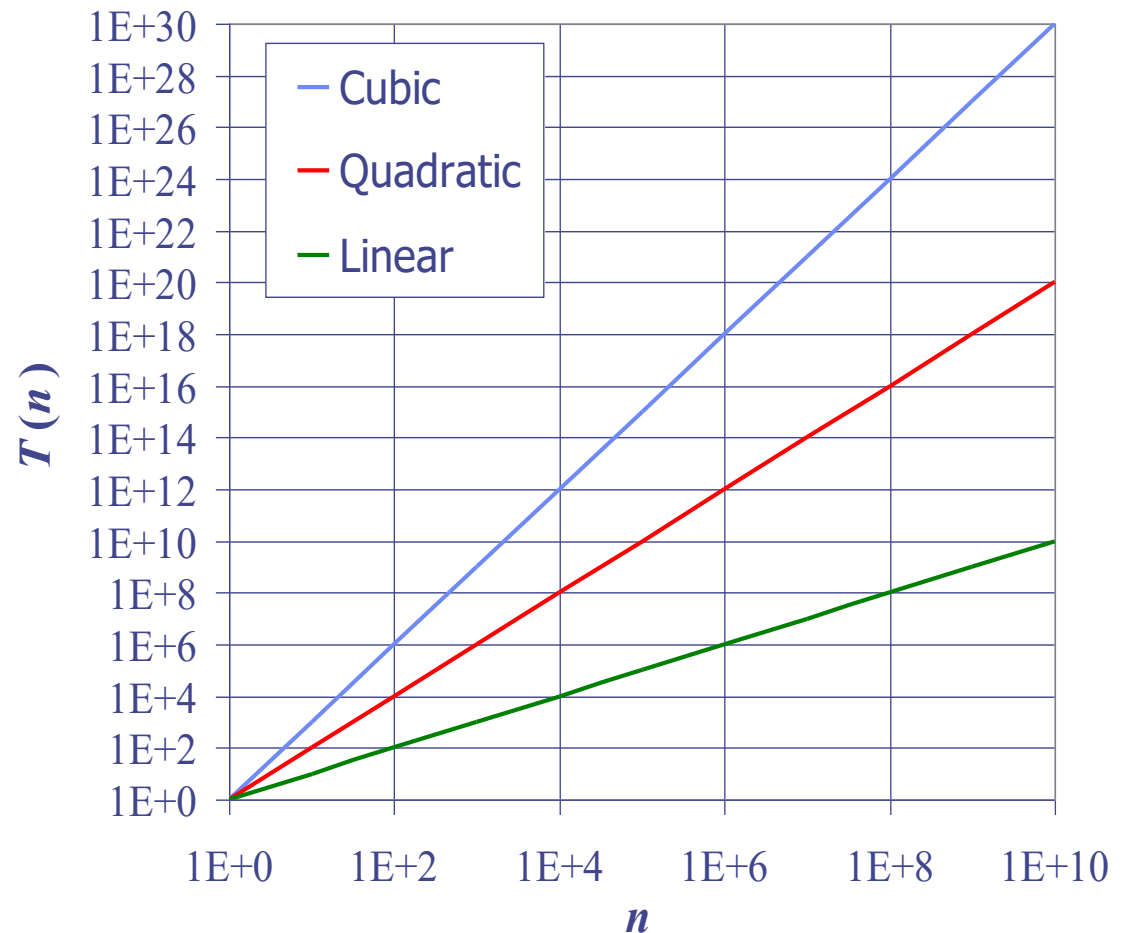
$n^2$  Superscripts and other  
mathematical formatting  
allowed

# Seven Important Functions (§3.3)

◆ Seven functions that often appear in algorithm analysis:

- Constant  $\approx 1$
- Logarithmic  $\approx \log n$
- Linear  $\approx n$
- N-Log-N  $\approx n \log n$
- Quadratic  $\approx n^2$
- Cubic  $\approx n^3$
- Exponential  $\approx 2^n$

◆ In a log-log chart, the slope of the line corresponds to the growth rate of the function



# Primitive Operations

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- ◆ Basic computations performed by an algorithm
- ◆ Identifiable in pseudocode
- ◆ Largely independent from the programming language
- ◆ Exact definition not important (we will see why later)
- ◆ Assumed to take a **constant amount of time** in the RAM model

- ◆ Examples:
  - Evaluating an expression
  - Assigning a value to a variable
  - Indexing into an array
  - Calling a method
  - Returning from a method

# Counting Primitive Operations (§3.4)

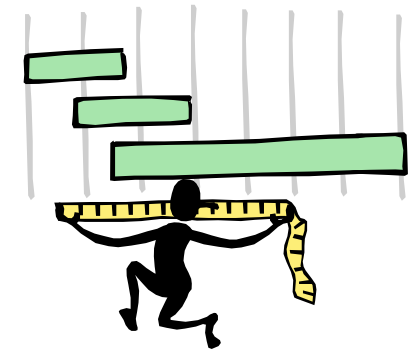
- ◆ By inspecting the pseudocode, we can determine the maximum number of primitive operations executed by an algorithm, as a function of the input size

<b>Algorithm</b> <i>arrayMax</i> ( <i>A</i> , <i>n</i> )	# operations
<i>currentMax</i> ← <i>A</i> [0]	2
<b>for</b> <i>i</i> ← 1 <b>to</b> <i>n</i> − 1 <b>do</b>	2 <i>n</i>
<b>if</b> <i>A</i> [ <i>i</i> ] > <i>currentMax</i> <b>then</b>	2( <i>n</i> − 1)
<i>currentMax</i> ← <i>A</i> [ <i>i</i> ]	2( <i>n</i> − 1)
{ increment counter <i>i</i> }	2( <i>n</i> − 1)
<b>return</b> <i>currentMax</i>	1
	<b>Total</b> 8 <i>n</i> − 2

# Estimating Running Time

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- ◆ Algorithm *arrayMax* executes  $8n - 2$  primitive operations in the worst case. Define:
  - $a$  = Time taken by the fastest primitive operation
  - $b$  = Time taken by the slowest primitive operation
- ◆ Let  $T(n)$  be worst-case time of *arrayMax*. Then
$$a(8n - 2) \leq T(n) \leq b(8n - 2)$$
- ◆ Hence, the running time  $T(n)$  is bounded by two linear functions

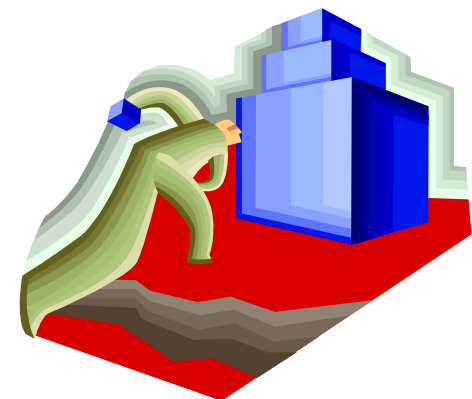




# Growth Rate of Running Time

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- ◆ Changing the hardware/ software environment
  - Affects  $T(n)$  by a constant factor, but
  - Does **not alter** the growth rate of  $T(n)$
- ◆ The linear growth rate of the running time  $T(n)$  is an intrinsic property of algorithm *arrayMax*



# Constant Factors

◆ The growth rate is **not** affected by

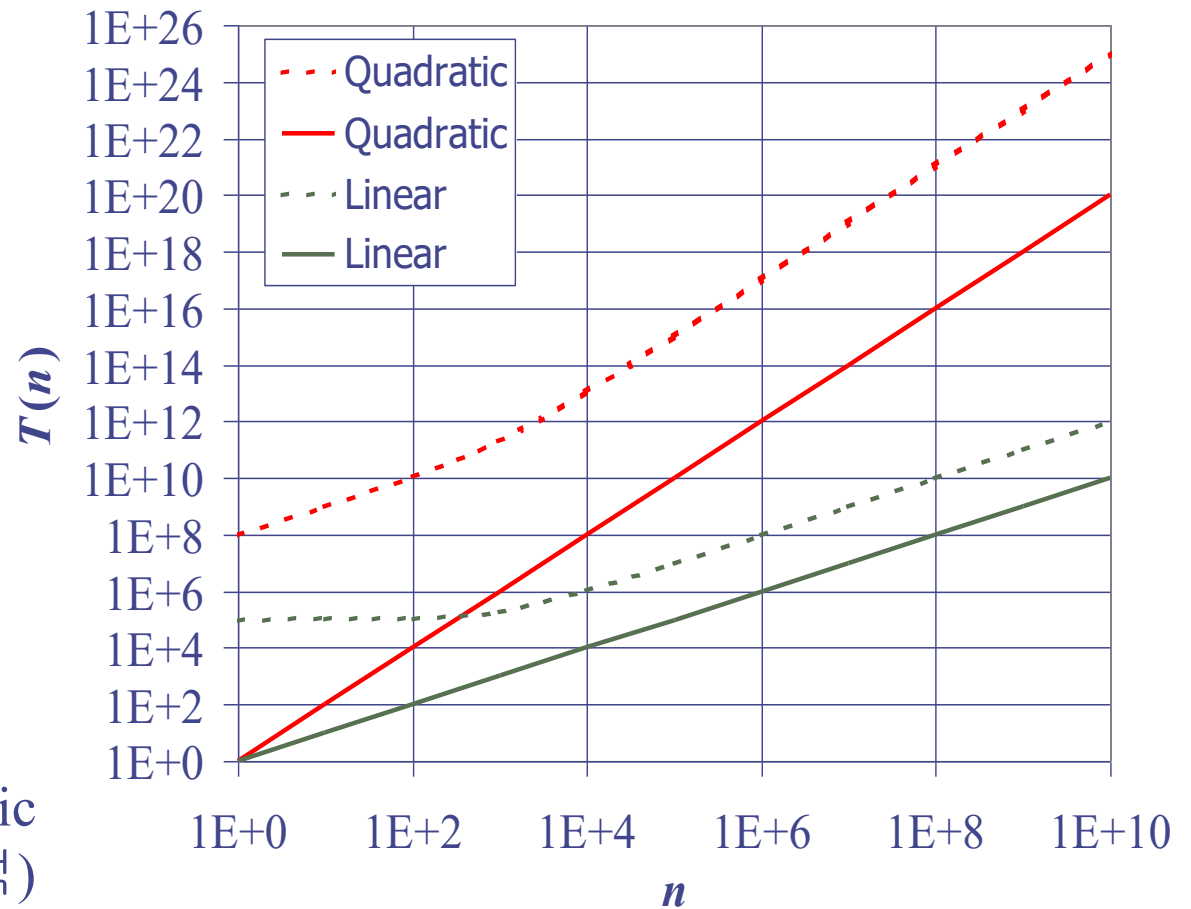
- constant factors or
- lower-order terms

◆ Examples

- $10^2n + 10^5$  is a linear function
- $10^5n^2 + 10^8n$  is a quadratic function

◆ We consider when  $n$  is sufficiently large

- We call this “Asymptotic Analysis” (점근적 분석)



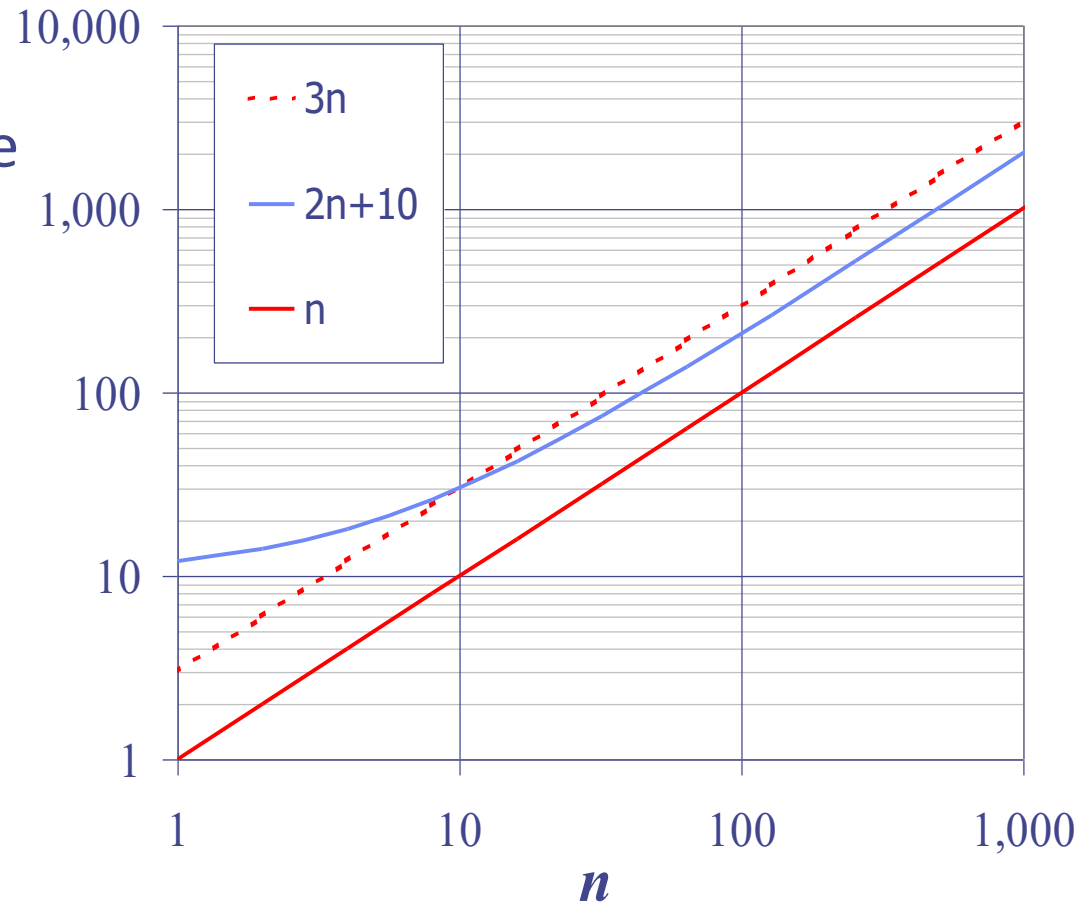
# Big-Oh Notation (§4.2.3)

◆ Given functions  $f(n)$  and  $g(n)$ , we say that  $f(n)$  is  $O(g(n))$  if there are positive constants  $c$  and  $n_0$  such that

$$f(n) \leq cg(n) \text{ for } n \geq n_0$$

◆ Example:  $2n + 10$  is  $O(n)$

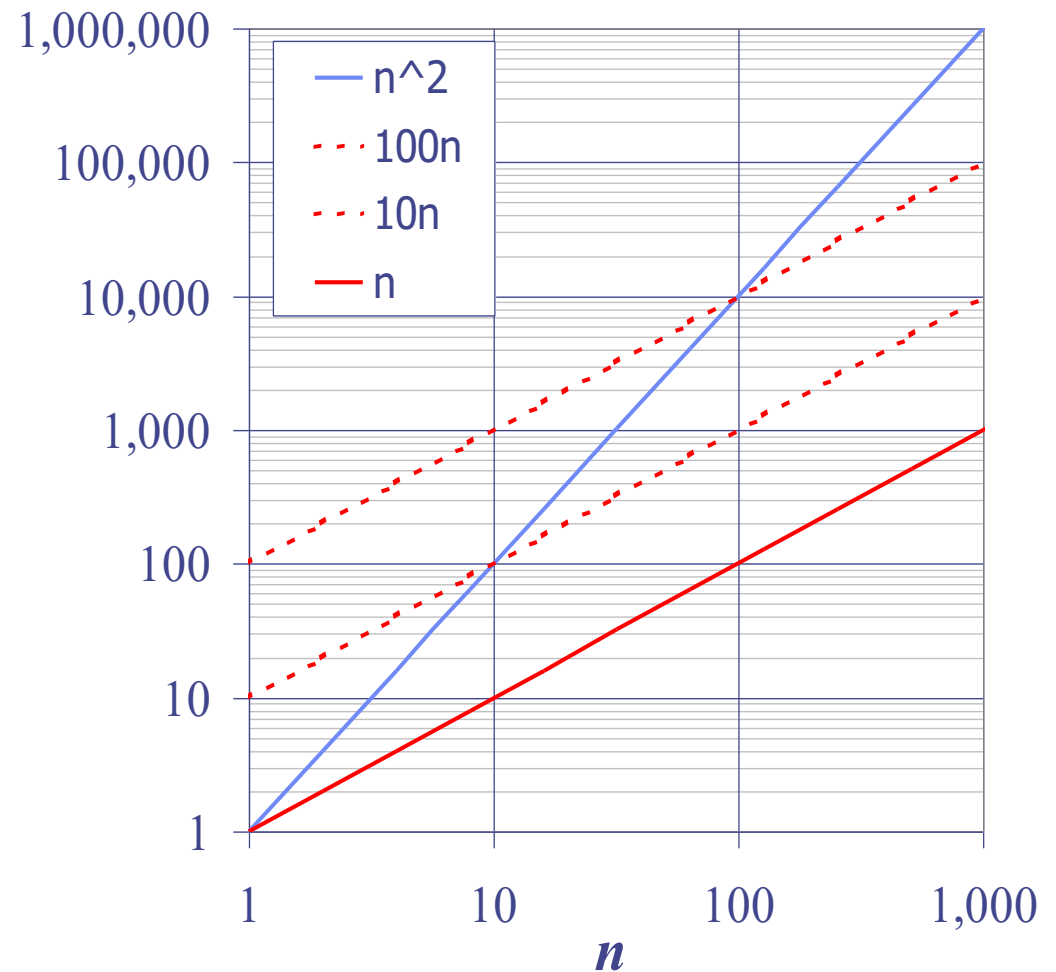
- $2n + 10 \leq cn$
- $(c - 2)n \geq 10$
- $n \geq 10/(c - 2)$
- Pick  $c = 3$  and  $n_0 = 10$



# Big-Oh Example

◆ Example: the function  $n^2$  is not  $O(n)$

- $n^2 \leq cn$
- $n \leq c$
- The above inequality cannot be satisfied since  $c$  must be a constant



# More Big Oh Examples



- $7n-2$

$7n-2$  is  $O(n)$

need  $c > 0$  and  $n_0 \geq 1$  such that  $7n-2 \leq c \cdot n$  for  $n \geq n_0$

this is true for  $c = 7$  and  $n_0 = 1$

- $3n^3 + 20n^2 + 5$

$3n^3 + 20n^2 + 5$  is  $O(n^3)$

need  $c > 0$  and  $n_0 \geq 1$  such that  $3n^3 + 20n^2 + 5 \leq c \cdot n^3$  for  $n \geq n_0$

this is true for  $c = 4$  and  $n_0 = 21$

- $3 \log n + 5$

$3 \log n + 5$  is  $O(\log n)$

need  $c > 0$  and  $n_0 \geq 1$  such that  $3 \log n + 5 \leq c \cdot \log n$  for  $n \geq n_0$

this is true for  $c = 8$  and  $n_0 = 2$

- **(Question)  $3 \log n + 5$  is  $O(n)$ ? Yes or No?**

# Big-Oh and Growth Rate

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- ◆ The big-Oh notation gives an **upper bound** on the growth rate of a function
- ◆ The statement “ $f(n)$  is  $O(g(n))$ ” means that the growth rate of  $f(n)$  is **no more than** the growth rate of  $g(n)$
- ◆ We can use the big-Oh notation to rank functions according to their growth rate

Which is possible?

	$f(n)$ is $O(g(n))$	$g(n)$ is $O(f(n))$
$g(n)$ grows faster	Yes	No
$f(n)$ grows faster	No	Yes
Same growth	Yes	Yes

# Big-Oh Rules

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- ◆ If  $f(n)$  is a polynomial of degree  $d$ , then  $f(n)$  is  $O(n^d)$ , i.e.,
  1. Drop lower-order terms
  2. Drop constant factors
  
- ◆ Use the smallest possible class of functions
  - Say “ $2n$  is  $O(n)$ ” instead of “ $2n$  is  $O(n^2)$ ”
  
- ◆ Use the simplest expression of the class
  - Say “ $3n + 5$  is  $O(n)$ ” instead of “ $3n + 5$  is  $O(3n)$ ”

# Asymptotic Algorithm Analysis

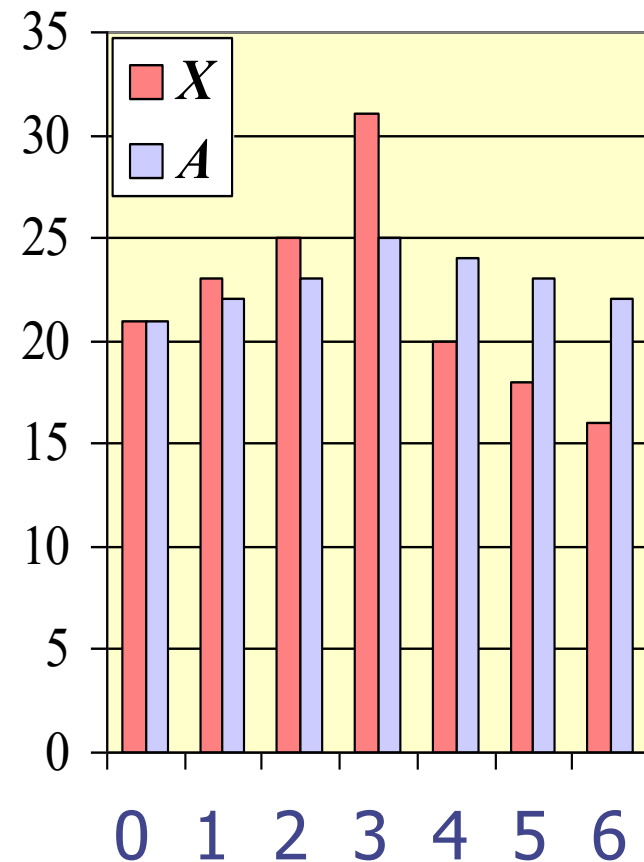
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- ◆ The asymptotic analysis of an algorithm determines the running time in big-Oh notation
- ◆ To perform the asymptotic analysis
  - We find the worst-case number of primitive operations executed as a function of the input size
  - We express this function with big-Oh notation
- ◆ Example:
  - We determine that algorithm *arrayMax* executes at most  $8n - 2$  primitive operations
  - We say that algorithm *arrayMax* “runs in  $O(n)$  time”
- ◆ Since constant factors and lower-order terms are eventually dropped anyhow, we can disregard them when counting primitive operations



# Computing Prefix Averages

- ◆ We further illustrate asymptotic analysis with two algorithms for prefix averages
- ◆ The  $i$ -th prefix average of an array  $X$  is average of the first  $(i + 1)$  elements of  $X$ :  
$$A[i] = (X[0] + X[1] + \dots + X[i]) / (i+1)$$
- ◆ Computing the array  $A$  of prefix averages of another array  $X$  has applications to financial analysis



# Prefix Averages (Quadratic)

- ◆ The following algorithm computes prefix averages in quadratic time by applying the definition

**Algorithm** *prefixAverages1*( $X, n$ )

**Input** array  $X$  of  $n$  integers

**Output** array  $A$  of prefix averages of  $X$       #operations

$A \leftarrow$  new array of  $n$  integers       $n$

**for**  $i \leftarrow 0$  **to**  $n - 1$  **do**       $n$

$s \leftarrow X[0]$        $n$

**for**  $j \leftarrow 1$  **to**  $i$  **do**       $1 + 2 + \dots + (n - 1)$

$s \leftarrow s + X[j]$        $1 + 2 + \dots + (n - 1)$

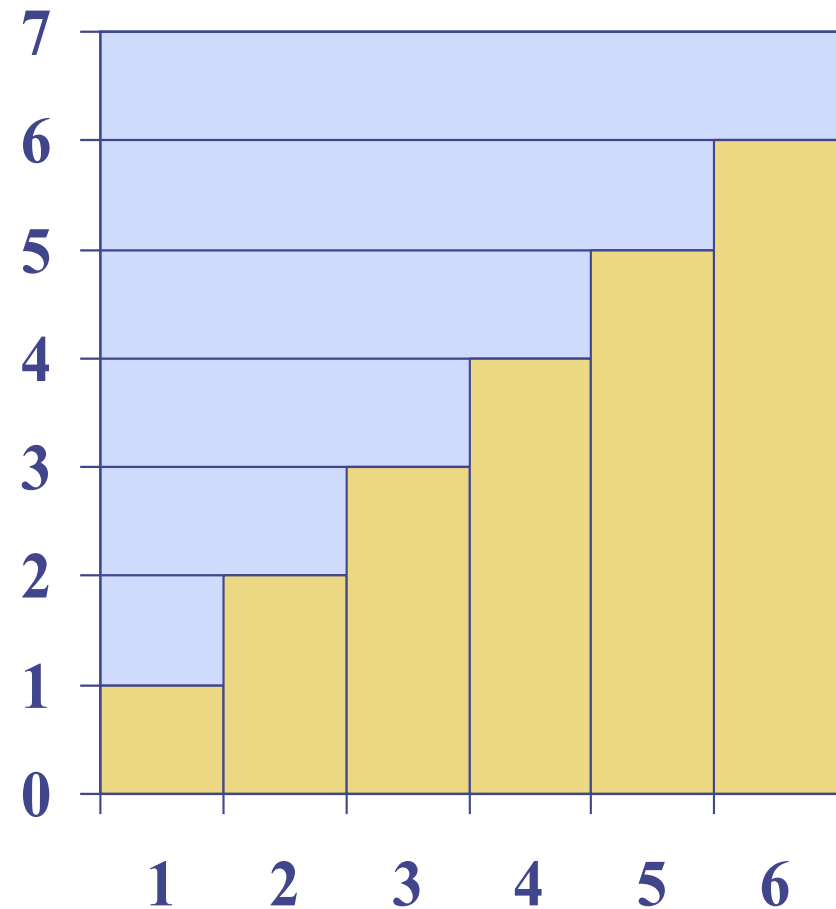
$A[i] \leftarrow s / (i + 1)$        $n$

**return**  $A$       1

# Arithmetic Progression

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- ◆ The running time of *prefixAverages1* is  $O(1 + 2 + \dots + n)$
- ◆ The sum of the first  $n$  integers is  $n(n + 1) / 2$ 
  - There is a simple visual proof of this fact
- ◆ Thus, algorithm *prefixAverages1* runs in  $O(n^2)$  time



# Prefix Averages (Linear)

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- ◆ The following algorithm computes prefix averages in linear time by keeping a running sum

<b>Algorithm</b> <i>prefixAverages2</i> ( $X, n$ )	
<b>Input</b> array $X$ of $n$ integers	
<b>Output</b> array $A$ of prefix averages of $X$	#operations
$A \leftarrow$ new array of $n$ integers	$n$
$s \leftarrow 0$	1
<b>for</b> $i \leftarrow 0$ <b>to</b> $n - 1$ <b>do</b>	$n$
$s \leftarrow s + X[i]$	$n$
$A[i] \leftarrow s / (i + 1)$	$n$
<b>return</b> $A$	1

- ◆ Algorithm *prefixAverages2* runs in  $O(n)$  time

# Another Example

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*Result*  $\leftarrow$  0; *m*  $\leftarrow$  1;

*for l*  $\leftarrow$  1 to *n*

*m*  $\leftarrow$  *m*\*2;

*for j*  $\leftarrow$  1 to *m* do

*result*  $\leftarrow$  *result* + *i*\**m*\**j*

# Math you need to review

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- ◆ Summations
- ◆ Logarithms and Exponents
  
- ◆ Proof techniques
- ◆ Basic probability
  - ◆ For randomized algorithms (later in this course)

- ◆ **properties of logarithms:**

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b(x/y) = \log_b x - \log_b y$$

$$\log_b x^a = a \log_b x$$

$$\log_b a = \log_x a / \log_x b$$

- ◆ **properties of exponentials:**

$$a^{(b+c)} = a^b a^c$$

$$a^{bc} = (a^b)^c$$

$$a^b / a^c = a^{(b-c)}$$

$$b = a^{\log_a b}$$

$$b^c = a^{c \cdot \log_a b}$$

# Relatives of Big-Oh

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## ◆ big-Omega

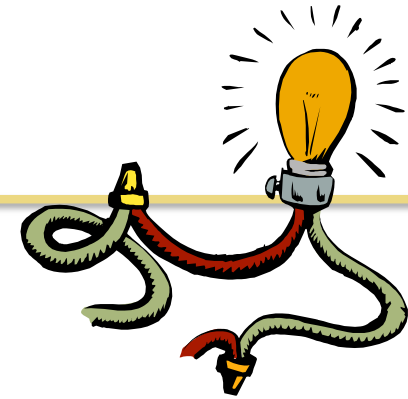
- $f(n)$  is  $\Omega(g(n))$  if there is a constant  $c > 0$  and an integer constant  $n_0 \geq 1$  such that  $f(n) \geq c \cdot g(n)$  for  $n \geq n_0$

## ◆ big-Theta

- $f(n)$  is  $\Theta(g(n))$  if there are constants  $c' > 0$  and  $c'' > 0$  and an integer constant  $n_0 \geq 1$  such that  $c' \cdot g(n) \leq f(n) \leq c'' \cdot g(n)$  for  $n \geq n_0$

# Intuition for Asymptotic Notation

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## Big-Oh

- $f(n)$  is  $O(g(n))$  if  $f(n)$  is asymptotically **less than or equal** to  $g(n)$

## big-Omega

- $f(n)$  is  $\Omega(g(n))$  if  $f(n)$  is asymptotically **greater than or equal** to  $g(n)$

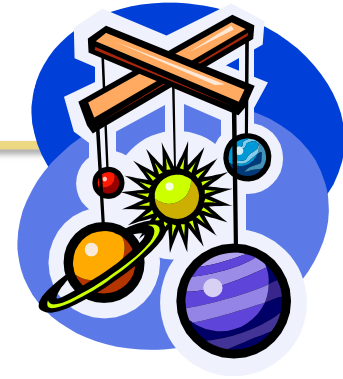
## big-Theta

- $f(n)$  is  $\Theta(g(n))$  if  $f(n)$  is asymptotically **equal** to  $g(n)$



# Examples (1)

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- $5n^2$  is  $\Omega(n^2)$

$f(n)$  is  $\Omega(g(n))$  if there is a constant  $c > 0$  and an integer constant  $n_0 \geq 1$  such that  $f(n) \geq c \cdot g(n)$  for  $n \geq n_0$

let  $c = 5$  and  $n_0 = 1$

- $5n^2$  is  $\Omega(n)$

$f(n)$  is  $\Omega(g(n))$  if there is a constant  $c > 0$  and an integer constant  $n_0 \geq 1$  such that  $f(n) \geq c \cdot g(n)$  for  $n \geq n_0$

let  $c = 1$  and  $n_0 = 1$

## Examples (2)

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- $5n^2$  is  $\Theta(n^2)$

$f(n)$  is  $\Theta(g(n))$  if it is  $\Omega(n^2)$  and  $O(n^2)$ . we have already seen the former, for the latter (for  $O(n^2)$ ) recall that  $f(n)$  is  $O(g(n))$  if there is a constant  $c > 0$  and an integer constant  $n_0 \geq 1$  such that  $f(n) \leq c \cdot g(n)$  for  $n \geq n_0$

Let  $c = 5$  and  $n_0 = 1$

# What do we want for our algorithms?

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- ◆ Prof. Yung Yi → A graduate student
  - “What is the **order** of your algorithm?”
  - Answer:  $n \log n$ ,  $n^2$ ,  $n^3$ ,  $2^n$
- ◆ Polynomial order
  - Generally fine.
  - Try to reduce the running time if above or equal to  $n^3$
- ◆ There are some problems for which there does NOT exist any polynomial-time algorithm (up to so far)
  - We say that they “NP-hard” or “NP-complete”
  - You will learn formalism for this in the algorithm class