Analysis of Complex Networks
Lecture 2: E-R Graph

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Contents

- What kind of graph models are we going to use to analyze a complex network?
- Could be many, but let's first consider the simplest one.
- ER Graph: Erdős-Rény Graph, also simply called a random graph.

ER Graph

- Denote the graph by $G(n,p)$, where n and p are parameters.

- Each edge is formed with probability $p \in (0,1)$ independently of every other edge, and n is the number of nodes.

- Let ξ_{uv} be a Bernoulli R.V. indicating the presence of edge between two nodes u and v, where u, v are some two nodes, i.e., $\xi_{uv} = 1$ with probability p and 0 with probability $1-p$.

- Then,

$$\mathbb{E}[\text{number of edges}] = \qquad\qquad = \qquad\qquad .$$

- Statistic properties of $G(n,p)$
  - Degree distribution?
  - Average path length?
  - Diameter?

ER Graph: Degree Distribution

- Let D be a R.V. representing the degree of a node.

- D is a ( ) R.V. with parameters ( ). Thus,

$$\mathbb{P}[D = d] = \qquad .$$

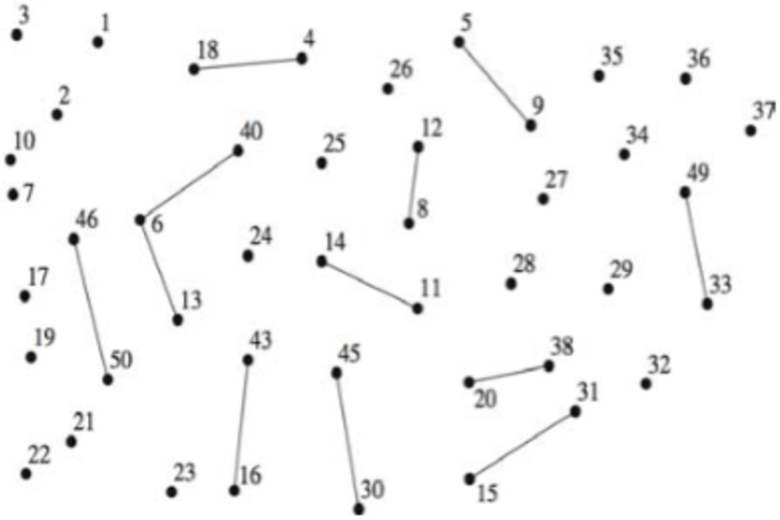
- If we keep the expected degree constant as $n \to \infty$, D is approximated by a ( ) R.V. with $\lambda =$, i.e.,

$$\mathbb{P}[D = d] = \qquad .$$

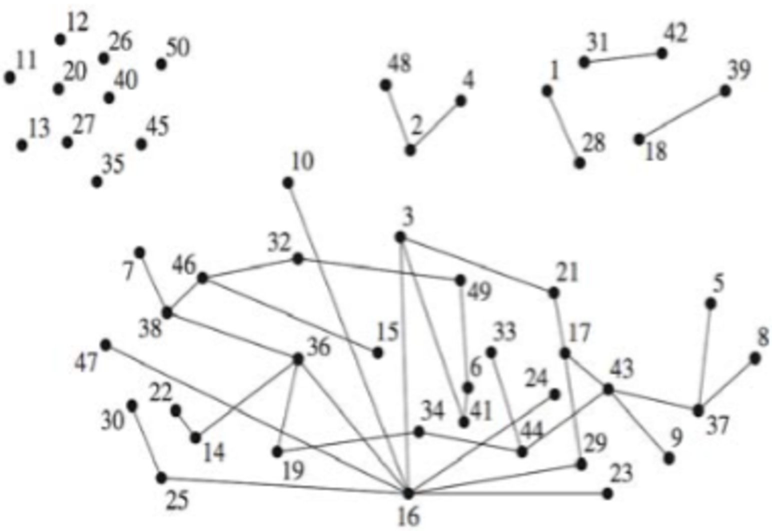
  Thus, ER graph is also called ( ).

Graphs with Different Parameters

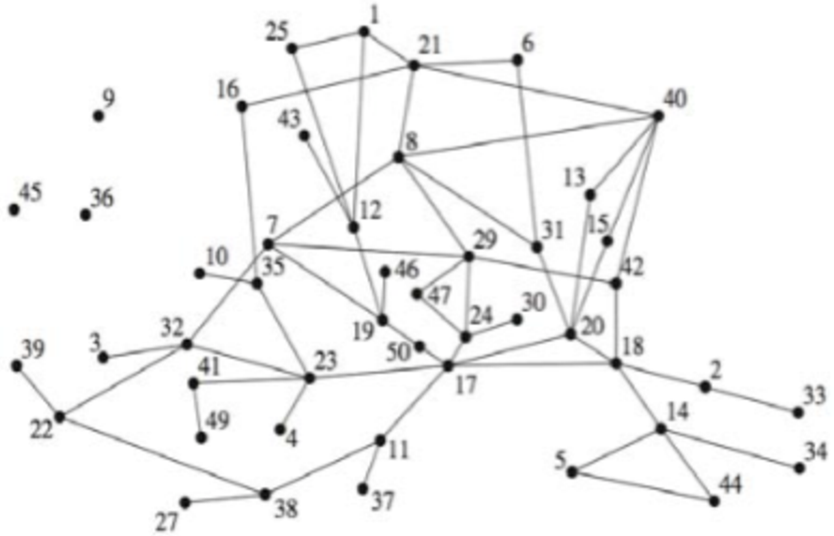
- $G(50, 0.01)$, A first component with more than two nodes



- $G(50, 0.03)$, Emergence of cycles



- $G(50, 0.05)$, Emergence of a giant component



- $G(50, 0.10)$, Emergence of connectedness