P_r: $\forall \varepsilon > 0$ $\Pr(A_n(\varepsilon)) \xrightarrow{n\to\infty} 0$ (Convergence in probability)

- $\forall \varepsilon > 0$ $\sum_n P(A_n(\varepsilon)) < \infty \longrightarrow$ a.s   정리 $\sum a_n < \infty$ ↓ ↑(X) $\lim_n a_n = 0$

↓ ↑X ↑ strong 한 조건

$\Pr(A_n(\varepsilon)) \xrightarrow{n\to\infty} 0$

B-C Lemma 의 활용 $X_n \to X$ a.s 하려면 보이는 (sufficient)

① 정의를 이용하기

② $\sum_n P(A_n(\varepsilon)) < \infty \longrightarrow P(A_n(\varepsilon)) \xrightarrow{n\to\infty} 0$

a.s

$\Pr(|X_n - X| > \varepsilon) = \frac{1}{n}$ (X)

$\sum \frac{1}{n} < \infty$ (X)

$= \frac{1}{n^2}$ (O) $\sum \frac{1}{n^2} < \infty$

주제: Gronwall's Lemma 증명

Mode of Convergence

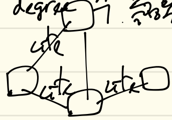
Converge in probability (O) → a.s (X)

체크: B.C test $\sum P(A_n(\varepsilon)) < \infty$ 가능할 경우 가능

Lecture 12 (chapter 7) Power laws via Preferential attachment

현상들~대한 분포 — X — 통계량

① 노드의 degree 수 (FB에서 친구의 수)
② 각 Youtube video의 랭킹 (각 video의 접속수)
③ citation network : degree 수 논문들이 얼마나 cite 됐는지



④ web page 접속수
⑤ biological network

"complex network"

평균 (first moment) : 중요정보
분산 : 평균에서 얼마나 멀어져 있는가?
불충분함

의미가 있을때 : 분포가 평균을 중심으로 잘 모여 있을 때
더 벗어나는 경우 : $\boxed{\text{평균에서 많이 멀어져 있다}}$ → 분산 higher moment (치우침) 어떻게 퍼져있는가

Degree ① 수백이 많은수 (x)
친구를 가지는 어떤 인물, 관찰자들이 엄청 많다 — "빈익빈 부익부"

"Degree 분포" : $\boxed{\text{빈익빈 부익부}}$

- ER graph
- small world

평균에 잘 모여있는

(Q) 그렇다면 이러한 "빈익빈 부익부" graph는 어디서 기인했을까?
⇒ 각 node들이 연결을 어떤 형태로 맺길래 → 이런 graph가 나올까?

(수학적)

★ 빈익빈 부익부 graph → 수학적인 quantification (degree 분포가 어떤 형태인)
↳ power law

★

** X, $p(x)$ $\boxed{p(x) \propto x^{-\alpha}}$ $\boxed{p(x) = C\cdot x^{-\alpha}}$ → for some constant α. → power law를 따를까

degree 분포가 power law를 따를까 → graph: power law graph

결론

← 비례상수 때문에 '변수의 법칙'을 표현할 수 있는가?

$p(x)$ $\alpha > 1$, $\boxed{p(x) = C\cdot x^{-\alpha}}$ → 상수끼리만 진행

$1<\alpha\leq 2$: 평균: infinite, higher: infinite

$2<\alpha<3$: 평균: finite, 분산: infinite

first moment, second moment

중심극한정리

$\alpha > 3$(???) higher moments: infinite

$\alpha > 2$: 평균은 finite, 분산: 크다 → 변수의 법칙 성립한다.

이제의 fundamental 한 질문: Algorithm graph의 연결형태 (현실세계에서 상식적으로) → power law degree distribution

① What is this?

② 수학적 결론, 실제에 적용

Barabasi → algorithm 제안: Preferential attachment → power law

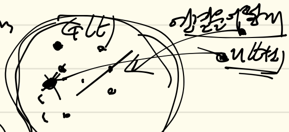
Preferential attachment (아주 상식적으로) (page 78 의 아래)

- A graph is grown over time $G(0), G(1), G(2), \ldots G(t)$ $\quad G_t = (N(t), E(t))$ random
- every step t, we add one new node random

at step $t+1$,

- Given ~~G(t)~~ $G(t) = (N(t), E(t))$, a new node $u(t+1)$ is added

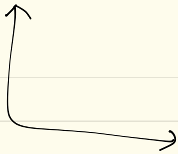
① with probability α, uniformly at random 으로 연결



② with probability $1-\alpha$, a node $v \in N(t)$ is selected with: $\dfrac{d_t(v)}{2E(t)}$ ⇒ degree가 많은것이 높은 확률로 붙는다.

너무 상식적

(Q) PA → powerlaw degree distribution



(과제) 과제 research problem : ① 수학적 결론의 표현?

매우 순진한 과정 | 엄밀하게 안 보인 가능성

예시 → (weak / strong)

연결고리 불충분?

무→유 (시간 변화) : Yule process

배수(?) | 1925 정보

~~Azuma~~ AY inequality

"Coupling"

power law distribution, heavy-tail distribution, scale-free network

(i) scale-free (scale-invariant)

$p(\underline{d})$: degree distribution ~ power law $= Cd^{-r}$

$$\frac{p(d)}{p(d')} = \frac{C\cdot d^{-r}}{C(d')^{-r}} = \left(\frac{d}{d'}\right)^{-r} \Rightarrow$$ d과 d'를 rescale해봐라 by any factor

$$\frac{p(md)}{p(md')} =$$ → 같다 (같은 값 유지)

$d \overset{\text{ratio}}{\longleftrightarrow} d'$:

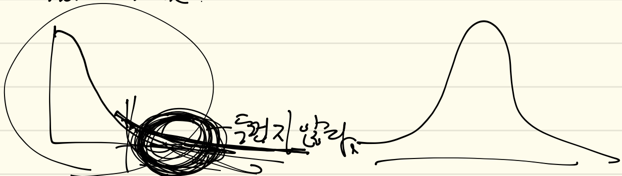
↓

md_md' : 변하지 않는다 (ratio)

"relative probabilities of different degrees depend only on their ratio, not their absolute size"

(ii) heavy-tail (tail이 두껍다)

long-tail



The distribution of RV X with distribution function F (CDF) is said to have "heavy (right) tail" if

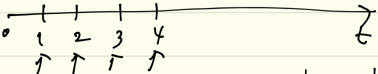
$$\lim_{x\to\infty} e^{\lambda x} P(X>x) = \infty \quad \text{for all } \lambda > 0$$

$$\Leftrightarrow \lim_{x\to\infty} e^{\lambda x} \bar{F}(x) = \infty$$

power-law 이라면 → heavy tail

$$e^{\lambda x} P(X > x) = e^{\lambda x} \int_x^{\infty} C \cdot t^{-\beta} \, dt = C e^{\lambda x} \left(\left[\frac{t^{-\beta+1}}{-\beta+1} \right]_x^{\infty} \right) \qquad \beta > 2$$

$$= C \ast e^{\lambda x} \left(- \frac{x^{-\beta+1}}{-\beta+1} \right) \xrightarrow{x \to \infty} \infty$$



node 1개씩 추가

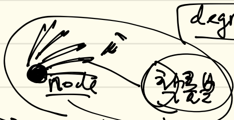
Markov chain (discrete time)

$\vec{X}(t) = [X_i(t)]_i$

any 상태: $X_i(t)$은 시간 t에서 degree가 i개인 node의 수 :

$\frac{X_i(t)}{t}$: 전체 node에서 degree가 i → 전체 node의 수 개인 node의 수

degree distribution



→ 이 Markov chain이 ergodic

time average

= ensemble average

i 늘리기 $(i > 1)$

$$\mathbb{P}\left(X_i(t+1) = X_i(t) + 1 \mid \vec{X}(t) \right)$$: 새로운 node가 degree $i-1$인 node에 붙을때

$$= \alpha \cdot \frac{X_{i-1}(t)}{N(t)} + (1-\alpha) \frac{(i-1) \cdot X_{i-1}(t)}{2E(t)} \quad \text{— ①}$$

→ α : random, $1-\alpha$: degree

$$\mathbb{P}\left(X_i(t+1) = X_i(t) - 1 \mid \vec{X}(t) \right)$$: 새로운 node가 degree i인 애랑 연결

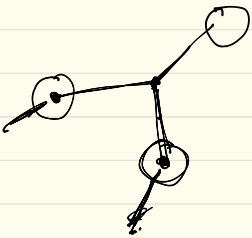
$$= \alpha \frac{X_i(t)}{N(t)} + (1-\alpha) \frac{i \cdot X_i(t)}{2E(t)} \quad \text{— ②}$$

$$\mathbb{P}\left(X_i(t+1) = X_i(t) \mid \vec{X}(t) \right)$$

$$= 1 - ① - ②$$

$i=1$인 경우 특별하다.

$P(X_1(t+1) = X_1(t) - 1 \mid \vec{X}(t)) = 0 \quad (X)$



$$P\left(X_1(t+1) = X_1(t) \mid \vec{X}(t) \right)$$

$$= \alpha \frac{X_1(t)}{N(t)} + (1-\alpha) \cdot \frac{1 \times X_1(t)}{2E(t)} \quad \text{— ③}$$

$$P\left(X_1(t+1) = X_1(t) + 1 \mid \vec{X}(t) \right) = 1 - ③$$

$\boxed{\text{Thm 7.1}}$ for all $i \geq 1$,

$$\frac{X_i(t)}{t} \xrightarrow{a.s.\ (t\to\infty)} C_i,$$

where $C_1 = \dfrac{2}{3+\alpha}$, $\quad \dfrac{C_i}{C_{i-1}} = \dfrac{\alpha + \frac{(1-\alpha)}{2}(i-1)}{1+\alpha + \left(\frac{1-\alpha}{2}\right) i}$, $i > 1$

⇒ degree가 큰 쪽에서는 powerlaw이다

(Note) 헷갈리지 말 것 $\boxed{\dfrac{X_i(t)}{t} \text{의 분포가 } (t\text{가 클 때}) \ C i^{-\beta}}$ weak하게 (i가 충분히 클 때) 예상

(해석) (powerlaw인지 관계)

$$C_j = C_1 \cdot \frac{C_2}{C_1} \cdot \frac{C_3}{C_2} \times \cdots \times \frac{C_j}{C_{j-1}} = C_1 \prod_{i=2}^{j} \frac{C_i}{C_{i-1}}$$

$$\frac{C_i}{C_{i-1}} = \frac{\alpha + \frac{(1-\alpha)}{2}(i-1)}{1+\alpha + \left(\frac{1-\alpha}{2}\right) i} = \frac{2\alpha + (1-\alpha)(i-1)}{2+2\alpha+(1-\alpha) i} = 1 - \frac{3-\alpha}{2+2\alpha+(1-\alpha)i}$$

$$= 1 - \frac{1}{i}\,\frac{3-\alpha}{\frac{2+2\alpha}{i} + (1-\alpha)} \qquad \frac{3-\alpha}{\frac{2+2\alpha}{i} + (1-\alpha)} = \frac{3-\alpha}{1-\alpha} + O(i^{-1})$$

$$= 1 - \frac{1}{i}\left(\frac{3-\alpha}{1-\alpha}\right) + O(i^{-2})$$

$$\log(C_j) = \log C_1 + \sum_{i=2}^{j} \log\left(1 - \frac{1}{i}\frac{(3-\alpha)}{(1-\alpha)} + O(i^{-2})\right)$$

($\log(1-x) \simeq x$ $\quad x$가 작으면)

($e^x \simeq 1-x \Leftrightarrow e^x - 1 \simeq x$)

$$\simeq \log C_1 + \sum_{i=2}^{j} \frac{1}{i}\,\frac{3-\alpha}{(1-\alpha)}$$

적분

$$\simeq \log C_1 - \beta \log j, \quad \boxed{\beta = \frac{3-\alpha}{1-\alpha}}$$

$$\boxed{C_j \simeq C_1 \cdot j^{\beta} \ (j\text{가 클 때})}$$

해석만 하고 싶어서

이문제를 어떻게 접근

① PA → node를 추가하는 MC degree i의 portion은 어느정도로 고정이 되지 않겠느냐?

$\frac{X_i(t)}{t} \xrightarrow{t\to\infty}$ stationary distribution

→ degree i의 state에 얼마나 자주 방문

state transition ~~dis~~ dynamics

$X_i(t)$ 와 $X_{i+1}(t)$ 관계식 복잡 c_i 와 c_{i+1}의 관계식으로 표현(?)

$$\boxed{\frac{X_i(t)}{t} \xrightarrow{a.s} c_i}$$

어려움 / 증명

→ a.s definition $\boxed{X_i(t) : \text{복잡한 process}}$

② 증명접근

$\frac{X_i(t)}{t} \xleftrightarrow{\text{관계}} c_i$

$\to \frac{\overline{X_i(t)}}{t}$

평균 (접근가능한식 비슷해)

$\hookrightarrow \leq \left|\frac{\overline{X_i(t)}}{t} - c_i\right| + \left|X_i(t) - \overline{X_i(t)}\right|$

재귀식이나 characterization 이용하여면

$\boxed{|X - E(X)| \leq M}$

A-H inequality

$\boxed{\text{Thm 7.2}}$ For all $\varepsilon > 0$, and for all $i \geq 1$

fix $t \geq 0$

$\overline{X_i(t)} = c_i t + o(t^{\varepsilon}) \iff \boxed{\frac{\overline{X_i(t)}}{t} = c_i + o(t^{\varepsilon - 1})}$

$t\to\infty$ → 0

$\boxed{\text{Lemma 7.3}}$ $\mathbb{P}\left(|X_i(t) - \overline{X_i(t)}| \geq M\right) \leq 2\exp\left(-\frac{M^2}{8t}\right)$ (A-H inequality)

$\boxed{\text{Proof of Thm 7.2}}$ A.S 증명은 Borel-Cantelli Lemma

$\left(A_n = \{|X - X_n| \geq \varepsilon\} \quad \sum_{n=0}^{\infty} A_n < \infty \longrightarrow X_n \xrightarrow{a.s} X \right)$

t

$\forall \varepsilon > 0$

↳ 이 증명과정을 따라감

$$P\left(\left|\frac{X_i(t)}{t}-c_i\right|\geq a(t)\right)\leq P_r\left(\left|\frac{X_i(t)}{t}-\frac{\overline{X_i}(t)}{t}\right|+\left|\frac{\overline{X_i}(t)}{t}-c_i\right|\geq a(t)\right)$$

$\Rightarrow$ a에 적절한 식 (t에 관한식)을 대입하여, $\sum < \infty$ $\quad = o(t^{\varepsilon-1})$ (Theorem 7-2)

$t\to\infty$ $a(t)\to 0$

$$=P_r\left(\left|\frac{X_i(t)}{t}-\frac{\overline{X_i}(t)}{t}\right|\geq a(t)-o(t^{\varepsilon-1})\right)\qquad a(t)=\boxed{\frac{4\sqrt{t\log t}}{t}}+o(t^{\varepsilon-1})$$

(Lemma 7.3)

$$\leq 2\cdot\exp\left(-\frac{\mu^2}{8t}\right)=2\cdot\exp\left(-\frac{16t\log t}{8t}\right)=2t^{-2}$$

μ자리

$t\to\infty$

$a(t)\to 0$

$$\sum_{t=1}^{\infty}2t^{-2}<\infty\longrightarrow\sum P\left(\left|\frac{X_i(t)}{t}-c_i\right|\geq a(t)\right)<\infty\qquad\blacksquare$$

$$\sum\frac{1}{t}=\infty,\quad\sum\frac{1}{t^2}<\infty$$

$\alpha>1$

$\boxed{\text{Thm 7.3}}$ $\frac{\overline{X_i}(t)}{t}$ 에 관련 이야기 $\overline{X_i}(t)$ "

$\underline{i=1}$

$$P\left(X_1(t+1)=X_1(t)\mid\vec{x}(t)\right)=\boxed{\alpha\cdot\frac{X_1(t)}{N(t)}+(1-\alpha)\cdot\frac{1\times X_1(t)}{2E(t)}}_{①}$$

$$P\left(X_1(t+1)=X_1(t)+1\mid\vec{x}(t)\right)=1-①$$

$$E(X_1(t+1))=E\left(E\left(X_1(t+1)\mid\vec{X}(t)\right)\right)=\sum_x p(X_1(t)=x)\cdot\left(\overbrace{x\left(\alpha\frac{x}{N(t)}+(1-\alpha)\frac{x}{2E(t)}\right)}^{A(t)}+(x+1)(1-A(t)\cdot x)\right)$$

$$=\sum_x p(X_1(t)=x)\left[x+1-A(t)x\right]\qquad\text{where}$$

$$=\overline{X_1}(t)+1-A(t)\cdot\overline{X_1}(t)\quad\therefore\ \boxed{\overline{X_1}(t+1)=\overline{X_1}(t)+1-A(t)\overline{X_1}(t)}$$

$\overline{X_1}(t)$ for a given t

$$\boxed{\bar{X}_1(t+1) = (1 - A(t))\bar{X}_1(t) + 1} \Longrightarrow \bar{X}_1(t) = C_1 t + o(t^{\varepsilon})$$

$$\Delta_1(t) = \bar{X}_1(t) - C_1 t$$

대입해보면 $\longrightarrow$ Homework (정확하게 풀이(x))

$$\Delta_1(t+1) = \Delta_1(t)\left(1 - \frac{\alpha}{N(t)} - \frac{1-\alpha}{2E(t)}\right) - \underbrace{C_1 t + 1 - C_1\left(\alpha + \frac{1-\alpha}{2}\right)}_{0} + O(t^{-1})$$

$C_1 = \frac{2}{3+\alpha}$ 약 1차이다름 합치고

$$\Delta_1(t+1) \leq \Delta_1(t) + O(t^{-1}) \leq O(\log t) \Rightarrow \Delta_1(t) = o(t^{\varepsilon}) \quad \forall \text{ for all } \varepsilon > 0$$

$$C_1 = \frac{2}{3+\alpha}$$

$(i > 1)$ 비슷한 방법 (as above) $\boxed{\text{Homework}}$ C_1, $\boxed{\frac{C_i}{C_{i-1}}}$ 관계

Lemma 7.3 For all $i, t \geq 1$, and all $M > 0$,

$$P\left(|X_i(t) - \bar{X}_i(t)| \geq M\right) \leq 2\exp\left(-\frac{M^2}{8t}\right) \Rightarrow \text{A-H inequality}$$

(Pf) Let i, t be fixed. 보내면서 graph가 어떻게 될지 안다

$$X_i(t) = f(v(1), v(2), \ldots, v(t)),$$

degree가 i인 node의 개수

where $v(s)$ is the node in $G(s-1)$ to which the node $u(s)$ attaches.

Also, Let $M(s) = E\left(X_i(t) \mid v(1), \ldots, v(s)\right)$, $s = 1, \ldots, t$

$$M(s-1) = E\left(X_i(t) \mid v(1), \ldots, v(s-1)\right)$$

$M(0) = E[X_i(t)]$ independent (x)

(i) $M(s)$ is a martingale
(ii) $\boxed{|M(s) - M(s-1)| \leq C}$, $C = 2$
$\Rightarrow$ A-H inequality

(i) easy: Doob martingale (ii) is true(?)

(iii) $\boxed{f\text{의 Lipschitz continuous 증명}}$ → $\boxed{\text{책 보강}}$ $|M(s) - M(s-1)| \le 2 \quad \forall s \le t$

$\boxed{\text{책 page 69}}$: Corollary 6.4 ($X_1, \ldots X_T$: independent)

반드시 봐야함

Coupling

(소문자: deterministic
대문자: random)

- Let s be fixed, and let the sequence $v(1), v(2), \ldots, v(s)$ be given
- Let another random variable $V'(s)$ of $G(s-1)$ be given, being distributed as the anchor node in $G(s)$, given $v(1), \ldots v(s-1)$

random

$v(1), v(2), \ldots, v(s-1), v(s) \mid V(s+1), V(s+2), \ldots, V(t)$ → $G(t)$

$G(1) \ldots G(s)$ ~~$G(t)$~~

($V'(s)$는 $v(s)$에 독립이다)

$V'(s), V'(s+1), \ldots, V'(t)$ → $G'(t)$

$G'(s), G'(s+1), \ldots G'(t+1)$ random

관계의 조건

"Coupling"

✓ 아무렇게나 (X) (DA에서 정해진대로 (X)) → 나만의 방법
(알 수 없다)

We will generate $\{V(s+1), \ldots, V(t)\}$, $\{V'(s), V'(s+1), \ldots, V'(t)\}$, such that the following properties are satisfied:

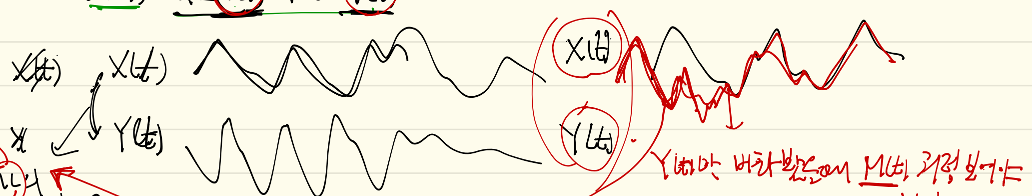
(resp. $V'(s+1), \ldots, V'(t)$)

(page 82) (i) • The distribution of $V(s+1), \ldots V(t)$ is that of the $(s+1)$th to t-th anchor nodes in the graph growth model that we consider, conditioned on the first s anchor nodes being $v(1), \ldots, v(s)$

(resp. $v(1), v(2), \ldots, v(s-1), V'(s-1)$)

(ii) For all $l \in s, \ldots t$, and any node u in the node set $G(l), G'(l)$, the degree $d_l(u)$ of u in $G(l)$ coincides with $d'_l(u)$ in $G'(l)$, unless $u = v(s)$ or $u = V'(s)$

보기 힘들다



~~Y~~ $M(t)$ 한 $M(t)$의 2개의 copy

$Y(t)$에만 변화를 줬을때 $M(t)$의 괴정 보여야 한다.

왜 이런 coupling을 하는가?

$M(s) = f(v(1), \dots, v(s)) = E[X_1(t) \mid v(1), \dots, v(s)]$

random 그래프

$G(t)$, $G'(t)$

$|M(s) - M(s-1)| \le$

$= \left| \sum_{v_{s+1}^t, v_s'^t} P(V_{s+1}^t = v_{s+1}^t, V_s'^t = v_s'^t) \left[f(v_1^t) - f(v_1^{s-1}, v_s'^t) \right] \right|$

$M(s)$: $(v(1), \dots, v(s), v(s+1), \dots, v(t))$

$M(s-1)$: $(v(1), \dots, v(s-1), v'(s), v'(s+1), \dots, v'(t))$

$(V(s+1), \dots, V(t))$

$G(t)$, $G'(t)$

$(V'(s+1), V'(s+2), \dots, V'(t))$

G' 그래프

$\le \sum_{v_{s+1}^t, v_s'^t} P(V_{s+1}^t = v_{s+1}^t, V_s'^t = v_s'^t) \left| f(v_1^t) - f(v_1^{s-1}, v_s'^t) \right|$

$V \le 2$

$v_1^t = v(1), v(2), \dots, v(s), \dots, v(t)$

$v_1^{s-1}, v_s'^t = v(1), v(2), \dots, v'(s), \dots, v'(t)$

$v(s)$

$v'(s)$

G, G'

≤ 2

남은일 (i)과 (ii)를 만족하는 coupling이 가능하다라는 것을 show

$\Rightarrow$ how? $\boxed{\text{Construction}}$ (Coupling 방법이 존재하다) (page 82의 half)

어떤 coupling $v(1) \dots, v(s-1)$ $v(s)$ G

$v'(s)$ G'

다른 것

$s+1$ coupling construction

At each step $l = s+1, s+2, \dots, t$

Define a Bernoulli random variable $Y_l = \begin{cases} 0 & \text{w.p. } \alpha \\ 1 & \text{w.p. } 1-\alpha \end{cases}$

Y_l, $l = s+1, \dots, t$

i.i.d

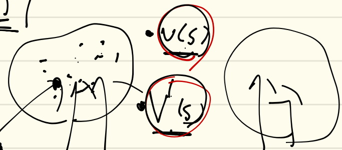
→ attachment 확률 모든 node uniformly at random

If $Y_\ell = 0$, choose an anchor node u (uniformly at random),

$V(\ell) = u \quad V'(\ell) = u$

If $Y_\ell = 1$, (각 node들에게 attachment 확률을 부여하고, 그 확률에 따라 선택)

① $u \notin \{V(s), V'(s)\}$



$$P(V(\ell) = u \mid Y_\ell = 1, V^{\ell-1}_{s+1}, V'^{\ell-1}_{s}) = \frac{d_{\ell-1}(u)}{2E(\ell-1)} \quad \text{and}$$

$$\boxed{V(\ell) = V'(\ell)}$$

② V(s)랑 V'(s)중에 붙는 확률 $(u, v) \in \{V(s), V'(s)\}$

$$P(V(\ell) = u, V'(\ell) = v \mid Y_\ell = 1, V^{\ell-1}_{s+1}, V'^{\ell-1}_{s}) = \frac{d_{\ell-1}(u)\cdot d'_{\ell-1}(v)}{2E(\ell-1)\,[d_{\ell-1}(V(s)) + d_{\ell-1}(V'(s))]}$$

$V(s)$	$V(s)$
$V(s)$	$V'(s)$
$V'(s)$	$V(s)$
$V'(s)$	$V'(s)$

(i) $$P(V(\ell) = u \mid V^{\ell-1}_{s+1}, V'^{\ell-1}_{s}) = \frac{\alpha}{N(\ell-1)} + (1-\alpha)\frac{d_{\ell-1}(u)}{2E(\ell-1)}$$

$u \notin (V(s), V'(s)) \to$ clear하다 immediately true

$u \in V(s)$ or $\boxed{v \in V'(s)}$

$$P(V(\ell) = V(s) \mid \quad) = P(V(\ell) = V(s), V'(\ell) = V(s)) + P(V(\ell) = V(s), V'(\ell) = V'(s))$$

$$= \frac{d_{\ell-1}(V(s))\,\cancel{(d_{\ell-1}(V(s)) + d_{\ell-1}(V'(s)))}}{2E(\ell-1)\,\cancel{(d_{\ell-1}(V(s)) + d_{\ell-1}(V'(s)))}} = \frac{d_{\ell-1}(V(s))}{2E(\ell-1)}$$

Similarly $Pr(V(\ell) = V'(s))$

Similarly $Pr(V'(\ell) = V(s))$, $Pr(V'(\ell) = V'(s))$ ....

(ii) $u \notin \{V(s), V'(s)\}$ degree coincides (L성질)