The Economic Effects of Sharing Femtocells

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***Abstract*—Femtocells are a promising technology for handling exponentially increasing wireless data traffic. Although extensive attention has been paid to resource control mechanisms, for example, power control and load balancing in femtocell networks, their success largely depends on whether operators and users accept this technology or not. In this paper, we study the economic aspects of femtocell services for the case of monopoly market, and aim to answer questions on operator's revenue, user surplus, and social welfare by considering practical service types and pricing strategies. We consider three user subscription services, that is, users can access only macro BSs (mobile-only), or deploy femto BSs in their house and open / exclusively use their femto BSs (open- / closed-femto). For pricing strategies, flat pricing and partial volume pricing are exploited. The main messages include the following: 1) open-femto service is beneficial to both users and providers; 2) in flat pricing, the impact on operator revenue of allowing or blocking the access of mobile-only users to open femto BSs is minor; and 3) compared with partial volume pricing, flat pricing is advantageous to the operator when users are sensitive to price.**

***Index Terms*—Femtocell, pricing, market model.**

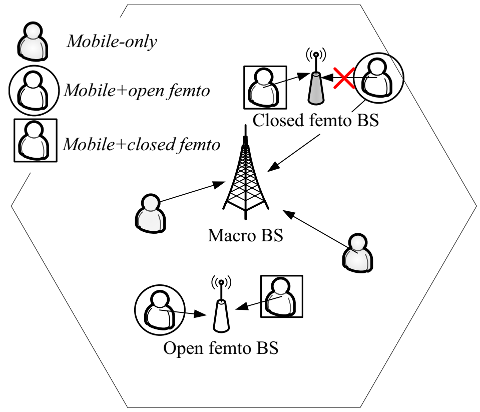

Fig. 1. Femtocell services: *mobile-only*, *mobile+open femto*, and *mobile+closed femto*

I. INTRODUCTION

A. Motivation

THE DEMAND for wireless data traffic is dramatically growing and the monthly demand has been forecasted to reach 6.3 EB [1] on 2015, a 26-fold increase over 2010 [1]. This unprecedented growth, which is driven by the introduction of smart mobile devices and the diversity of multimedia applications, throws up both challenges and opportunities for technical and business communities. In order to cope with this growing demand, many capacity enhancement solutions have been proposed. These include the system-wide upgrade to the 4G infrastructure, for example, LTE and WiMax, mainly by adopting enhanced physical layer technologies [2] or ad-hoc solutions such as offloading to WiFi [3], [4]. However, more dominant factor for capacity increase to handle traffic explosion in cellular systems is efficient frequency spatial reuse by reducing the cell sizes, e.g., micro, pico, and femto cells [5]. The key difference between micro/pico and femto cells largely lies in who deploys and controls the cell: micro/pico cell deployment is driven by operators, whereas users of femtocells tend to individually decide on deployment.

The femtocell technology, configuring a very small cell and the residential broadband backhaul, is economically attractive because it can achieve high spectral efficiency at a viable cost. Many researchers that have worked on femtocells have focused on technical issues [6]–[14], for example, spectrum sharing between macro and femtocells, interference management through power control, or handoffs and association for load balancing. However, limited attention has been paid to economic and business aspects, which is yet another important factor in the success of femtocell technology. Recently, Shetty and Walrand [15] proposed an economic framework and analyzed the economic impacts of adopting femtocells on the providers' revenue[2], which inspired our work. After that, the authors in [16] investigates the economic value of femtocells in view of spectrum allocation algorithms.

However, an important issue has remain under-explored: *openness of femtocells*. Since femtocells are typically installed in personal indoor environments, their capacity of femtocells tends to be higher than that of macrocells. Thus, it is expected that the utilization of the femtocells seems to be relatively low only with the femto owners' traffic. Therefore, it may be economically beneficial to users and providers to allow "guest" users to utilize the femto BSs that are open. However, it is far from clear how beneficial the open femtocell service is, depending on the factors, which have significant impact on the effect of femtocells on coping with mobile data explosion.

Manuscript received 10 March 2011; revised 1 September 2011. This research was supported by the KCC (Korea Communications Commission), Korea, under the R&D program supervised by the KCA (KCA-2011-11913-05004), the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MEST) (No. 2009-0075757), and MKE/KEIT (No. KI001865). Part of this work has been published at the proceedings of IEEE Infocom Mini-conference, 2011.

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Digital Object Identifier 10.1109/JSAC.2012.120409.

[1] 1EB or 1 exabyte is 10^{18} bytes.

[2] We use 'operator' and 'provider' interchangeably throughout this paper.

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B. Summary

We propose an analytical market model between the operator and users to understand the economic impact of sharing femtocells, whose major features are summarized as follows:

Service. As illustrated in Fig. 1, we consider three subscription services for accessing the network, namely *mobile-only*, *mobile+open femto*, and *mobile+closed femto*[3]. As the names imply, the users of *mobile-only* services can access only macro BSs, whereas the users of *mobile+open femto* and *mobile+closed femto* services have access to macro BSs as well as femtocells. The users of *mobile+closed femto* services exclusively use their femto BSs, whereas those of *mobile+open femto* services open their femto BSs which can be shared by other users.

Policies for sharing open femto BSs. Operators can choose one of two policies for users of *mobile+open femto* to share their femto BSs: *open-to-all* and *open-to-femto*. Users of *mobile-only* can access open femto cells under the *open-to-all* policy, but not the *open-to-femto* policy, which are open only to the users subscribing to the femto service.

Pricing. We consider two pricing schemes: *flat* and *partial volume pricing*. In both pricing schemes, a fixed service fee is charged for accessing femto BSs. This is because there is a practical concern that a complex metering may not be possible in femto BSs. The difference between two schemes lies in that in partial volume pricing users pay the service fee in proportion to the amount of the service by macro BSs, whereas in flat pricing the fixed fee is also charged for using macro BSs. The current femto service market trends support our choice of flat femto pricing. According to [17], for example, Vodafone in Spain offers a monthly pay plan and other providers such as Sprint and Docomo monthly charge the cost of BS equipment. KDDI and Softbank in Japan provide the femto services without additional charge. Verizon and AT&T in the US have a similar pricing scheme to the Japanese companies with minor difference in that they charge only once when the femtocell BS is installed.

Metric and market types. The major metrics are users' *surplus*, operator's *revenue*, and *social welfare*. We consider a monopoly market, where a single operator dominates and fully controls the market price to maximize its own revenue. Users simply follow the operator's price control and select the service that optimizes personal use.

The main messages of this paper are summarized as follows:

1) It is beneficial to both providers and users to have open-femto BSs rather than just closed ones. The difference between two services tends to grow as the number of users and the coverage of femto BSs increases.

2) The impact of open policies is minor in flat pricing. The differences to users and the provider are not significant whether or not the provider limits the access of femto BSs to *mobile-only* users. The subsidy for incentivising the femto users to open their femto BSs does not have to be large, and only needs to be approximately 10%-20% of the price of the *closed-femto* service.

3) The provider can achieve higher revenue with the flat pricing than it can with partial volume pricing when users are price-sensitive. However, these differences are significant with less sensitive users.

[3]We simply use *open-femto* and *closed-femto* to refer to *mobile+open femto* and *mobile+closed femto*, respectively.

TABLE I
SUMMARY OF MAJOR NOTATION (EXTERNAL PARAMETERS AND SYSTEM VARIABLES ARE SEPARATED)

External Parameter	*Description*
N	Number of users per one macro BS
$\gamma, \bar{\gamma}, \theta$	user type, max. of user type, price sensitivity
β	fraction of a femto BS's coverage
δ_o	probability that a user is outside
C_M, C_F	capacities of macro and femto BSs
C_C, C_O	capacities of closed and open femto BSs
η	femto capacity reduction factor
c_f	maintenance cost of a femto BS
q_o	fraction of coverage of all open femto BSs
U_j, Φ_j	expected utility and service fee of service type j
$\boldsymbol{\alpha} = (\alpha_m, \alpha_o, \alpha_c)$	user subscription ratios
R, S, W	revenue, user surplus, social welfare

C. Related Work and Organization

The economic aspects of access networks have been addressed by many researchers [18]–[22]. The paper [18] performed financial analysis of femtocell networks and claimed that there could be significant cost savings using a femtocell network. Shetty et al also showed the benefit of femtocell using a mathematical model [15]. However, these work did not address openness of femtocell. Pricing in Wi-Fi networks has been addressed in [19], [20]. In [19], the authors studied the economic incentives of WiFi network operators connecting to ISPs. The paper [20] studied the economic interaction between the WiFi and WiMax network providers.

The openness of access network is addressed in Wi-Fi based networks. In Korea, one of major provider, LG U+, announced a plan to open WiFi APs at homes for other guest users to extend WiFi connectivity [21]. FON [4] is another good example of open WiFi APs, where a user can use any WiFi APs of FON with no charge, if they allow other users' access subscribing to FON. The paper [22] dealt with business dynamics of open Wi-Fi networks. They claim that the evolution of such network depends on many factors such as initial coverage, subscription fee and user preference. Our work focuses on the economic aspect of users' willingness to choose and/or open the femto service. Note that we do not intend to compare the femto service with other access solutions such as operator-deployed pico/micro cells or FONs.

The rest of this paper is organized as follows: In Section II, we describe the system model. Sections III and IV provide the economic analysis for flat and partial volume pricing schemes, followed by the the numerical results in Section V. We conclude the paper in Section VI.

II. MODEL

A. System Model

Consider a wireless network consisting of macro and femto BSs, where N users/macro-cells are served by a monopoly

[4]http://www.fon.com.

operator. We assume a simple model of BSs, that is, macro and femto BSs provide the fixed capacities C_M and ηC_F, where C_F is the "pure" capacity of a femto BS and $\eta \in (0,1]$ is an interference factor. The value of η depends on spectrum sharing and femto open policy, whose details will be discussed in Section II-C.

Users are always guaranteed to be under the coverage of a macro BS, but not of a femto BS. We also assume that femtocell equipment is identical and that the coverage size of a femto BS is the fraction β of that of a macro BS. We do not consider the handover effects on the users, and assume that each user already has a backhaul connection based on residential Internet service, e.g., DSL (Digital Subscriber Line). We adopt this simple model to purely focus on the economic aspects of the system and to enable meaningful analysis.

Note that the instantaneous BS capacities and the volume of users' delivered data may largely depend on factors such as resource allocation mechanisms, and other factors (e.g., power control policy, channel conditions, user's distance to BS). However, simplifying the model does not overly change the key messages and insights because we focus on economic benefits and pricing, which typically take effect over a longer time-scale.

B. Monopoly Operator and Services

As mentioned in Section I, the operator provides three services, namely, *mobile-only*, *mobile+open femto*, and *mobile+closed femto*, and two femto open-policies: *open-to-all* and *open-to-femto*. We use $\{m, o, c\}$ to indicate these service types.

The operator charges $\phi_l^L(x)$ for generating traffic rate x in the BS type L to the user subscribing to service $l \in \{m, o, c\}$. The index L in the charging function aims to show the dependence of charging by the serving BS type. We consider two types of tariffs: *flat pricing* and *partial volume pricing*. We use $\{M, O, C\}$ to index to macro, open-femto, and closed-femto BSs.

In flat pricing, users' payments are constant regardless of data usage, that is, for any BS type $L \in \{M, O, C\}$,

$$\phi_l^L(x) = p_l, \ \ l \in \{m, o, c\}, \tag{1}$$

where p_l is the constant charge for service $l \in \{m, o, c\}$.

In partial volume pricing, users pay p_v^M per unit data rate when they are served by macro BSs, whereas they pay a fixed service fee p_m, p_o, p_c for using femto BSs. This hybrid setup is motivated by the practical reasons that low-cost femto BSs may not be appropriately equipped for complex per-data operations. Recall that operators do not use volume pricing for femto services [17]. Thus, for all $l \in \{m, o, c\}$, the pricing structure is represented as follows:

$$\begin{aligned} \phi_l^M(x) &= p_v^M x + p_l, && (2)\\ \phi_l^L(x) &= p_l, \ L \in \{O, C\}, && (3)\\ p_m &= 0, && (4) \end{aligned}$$

where $p_m = 0$ is due to the fact that (pure) volume pricing is applied when *mobile-only* users access macro BSs but cannot access femto BSs.

C. Capacity and Interference Model

The interference between macro and femto BSs depends on a spectrum sharing policy and service types. Under the separate carriers in which femto and macro BS do not share the spectrum, we can assume that the macro and femto capacities are fixed to be C_M and C_F. However, when they share the carriers, e.g., *partially shared carriers* [14], the "actual" capacities differ depending on the type of the femto BSs. In the case of closed femto, macro users around the closed femto BS can interfere the femto BS, possibly degrading the femto capacity. However, if a femto cell is open, mobile-only users can handoff to the open femto BS. To reflect this, we introduce the interference factor η and model a femto BS' capacity as:

$$\begin{aligned} C_C &= \eta C_F && \text{closed femto BS,}\\ C_O &= C_F && \text{open femto BS,} \end{aligned}$$

where C_F is again the "pure" femto BS capacity, C_C and C_O are "actual" capacities of the femto BS under closed and under open-femto policies, respectively, $\eta \in (0,1]$. We assume that the macro capacity C_M is not affected, which can be justified from the fact that interference mitigation scheme can be employed, e.g., *partially shared carriers* [14].

D. Users

To model the behavior of a user, we adopt an iso-elastic utility function [5] $u(x;\gamma)$, given by

$$u(x;\gamma) = \gamma x^\theta, \tag{5}$$

where x is traffic, γ is a user type value, and θ is an elasticity parameter. The utility function is an increasing concave function of traffic volume x, but with a decreasing marginal payoff. The user type parameter γ is introduced to model different willingness to pay. As the higher γ users have the more payoff for the same data-rate, they can afford more to subscribe to a service. We assume γ is uniformly distributed between $[0, \gamma_{max}]$, $\gamma_{max} > 0$. The parameter θ is closely related to *elasticity of demand*, that is, the percent change of demand to the percent change of price. A higher value of elasticity means more changes in demand to the price change. It is known that the elasticity for the given utility function $u(x;\gamma)$ is $\frac{1}{1-\theta}$. The iso-elastic function for data traffic is used in [23]–[25].

As the service rates are dissimilar for macro, open and closed femto BSs, we introduce *expected* utility and *expected* service fee functions for service type l, U_l, and Φ_l, as follows:

$$\begin{aligned} U_l(\boldsymbol{x};\gamma) &= \mathbb{E}\Big[u(x^L;\gamma)\Big] = \gamma \sum_{L\in\{M,O,C\}} (x^L)^\theta \pi_l^L, && (6)\\ \Phi_l(\boldsymbol{x}) &= \mathbb{E}\Big[\phi^L(x^L)\Big] = \sum_{L\in\{M,O,C\}} \phi_l^L(x^L)\pi_l^L. && (7) \end{aligned}$$

where π_l^L is the fraction of time or probability that users of service type l use a type L BS to get a service, and $\boldsymbol{x} = (x^M, x^O, x^C)$ is a vector that represents the traffic volume generated in each type of BS.

[5]An utility function $U(x)$ is said to be *iso-elastic* if for all $k > 0$, $U(kx) = f(k)U(x) + g(k)$ for some functions $f(k), g(k) > 0$.

TABLE II
PROBABILITY π_l^L FOR OPEN-TO-FEMTO AND OPEN-TO-ALL POLICIES

	Open-to-femto			*Open-to-all*		
	M	O	C	M	O	C
m	1	0	0	$\delta_o(1-q_o)+\delta_i$	$\delta_o q_o$	0
o	$\delta_o(1-q_o)$	$\delta_o q_o+\delta_i$	0	$\delta_o(1-q_o)$	$\delta_o q_o+\delta_i$	0
c	$\delta_o(1-q_o)$	$\delta_o q_o$	δ_i	$\delta_o(1-q_o)$	$\delta_o q_o$	δ_i

Then, the net-utility $\tilde{U}_l$ of service type l is given by

$$\tilde{U}_l(\boldsymbol{x};\gamma) = U_l(\boldsymbol{x};\gamma) - \Phi_l(\boldsymbol{x}),\ \ l \in \{m,o,c\}. \tag{8}$$

Users move and connect to different types of BSs over time. Users achieve different data rates, which also depends on the service type. Under our system model, when there are n *open-femto* users, the fraction of area q_o covered by the open femto BSs is given by:

$$q_o \triangleq 1-(1-\beta)^n. \tag{9}$$

Users' average mobility statistics are assumed to be equal. This is denoted by δ_i, which is the probability of being "inside," where $\delta_o = 1-\delta_i$. To users of femto services, δ_i corresponds to the fraction of time that they are under the coverage of their own femto BSs. The *mobile-only* users rely on macro BSs even when they are inside because of the absence of their own femto BSs. We ignore the possibility that the *mobile-only* users utilize neighboring femto BSs when they are inside for simplicity. When users are outside, they can access either a macro or an open-femto BS. They access an open-femto BS with a probability q_o or a macro BS with a probability $1-q_o$.

Table II shows $(\pi_l^L : l \in \{m,o,c\}, L \in \{M,O,C\})$ under different open policies. Under the *open-to-femto* policy, *mobile-only* users cannot access open femto BSs and can only access macro BSs, as shown on the first line. *Open-femto* users access open femto BSs with a probability $\delta_o q_o+\delta_i$ and macro BSs with probability $\delta_o(1-q_o)$. The *closed-femto* user case is shown in a similar manner. Under the *open-to-all policy*, even *mobile-only* users can access the open femto BSs.

E. Operators and Regulators

According to the user type γ and charging schemes, a user selects a service type and decides on the data demand. Let $\boldsymbol{\alpha} = (\alpha_l : l \in \{m,o,c\})$ be the vector of user fractions subscribing to each service type l. The service type and traffic rate vector of the user type γ are denoted by $l^*(\gamma)$ and $\boldsymbol{x}(\gamma)$, respectively. Thus, operator revenue (R), social welfare (W), and user surplus (S) are computed as

$$R = \int \Phi_{l^*(\gamma)}(\boldsymbol{x}(\gamma))Nd\gamma - (\alpha_o+\alpha_c)Nc_f, \tag{10}$$

$$W = \int U_{l^*(\gamma)}(\boldsymbol{x}(\gamma);\gamma)Nd\gamma - (\alpha_o+\alpha_c)Nc_f, \tag{11}$$

$$S = \int \tilde{U}_{l^*(\gamma)}(\boldsymbol{x}(\gamma);\gamma)Nd\gamma = W-R, \tag{12}$$

respectively, where c_f is the cost of a femto BS for the service provider.

III. Flat Pricing Market

A. Market Model

We first consider a market model under the flat pricing scheme. In this market model, the operator decides on the price vector $\boldsymbol{p} = (p_j : j \in \{m,o,c\})$ in order to maximize the revenue R by solving the following problem:

$$\textbf{Provider}: \qquad \max_{p_m,p_o,p_c\geq 0} R. \tag{13}$$

In the flat pricing scheme, the revenue in (10) is simplified to

$$R = N\Big(\sum_j p_j\alpha_j - c_f\cdot(\alpha_o+\alpha_c)\Big). \tag{14}$$

The subscription ratios $\boldsymbol{\alpha}$ vary by price. We assume that users are selfish and try to maximize individual (expected) utility. Thus, a user of type γ selects the service $j^*(\gamma)$ that maximizes his or her net-utility:

$$\textbf{User}: \qquad j^*(\gamma) = \arg\max_{j\in\{m,o,c\}} \tilde{U}_j(\boldsymbol{x};\gamma), \tag{15}$$

when his or her maximum net-utility is *positive,* and he or she does not select any service, otherwise. This market model can be modeled as a two-stage sequential game, where the operator determines the price vector to maximize the revenue in the first stage, and then, users select one of the services (or exit from the market) according to the price vector provided by the operator.

B. Traffic

We assume that users are saturated and have sufficient data to transmit whenever possible. The (average) amount of data generated by each user depends on service type, capacities C_M, C_O, and C_C, and the scheduling discipline of BSs for competitive users. In particular, we simply assume that a BS serves its served users equally [6]. Under this fairness assumption, the average service rate of a user served by macro BSs is inversely proportional to the number of users in a macro BSs, given by

$$x^M = C_M/(1+\sum_{j\in\{m,o,c\}} \pi_j^M\alpha_j N), \tag{16}$$

where the denominator in (16) corresponds to the total number of users in a macro BS, whereas a user is in the macro BS's service. Similarly, the service rates for users with open and closed femto BSs are given by

$$x^O = C_O/(1+\sum_{j\in\{m,o,c\}} \pi_j^O\alpha_j N/\alpha_o N), \tag{17}$$

$$x^C = C_C, \tag{18}$$

respectively, where note that $\pi_j^O\alpha_j N/\alpha_o N$ is the average number of users visiting an open femto BS.

[6] in view of long term average, users have the same data rate when their mobility patterns are homogeneous.

C. Equilibrium

User subscription ratios $\boldsymbol{\alpha} = (\alpha_j : j \in \{m, o, c\})$ are a function of price level $\boldsymbol{p}$ and data rate $\boldsymbol{x}$. Let γ_i be a point $\tilde{U}_i(x;\gamma_i) = 0$ for all $i \in \{m, o, c\}$ and γ_{ij} be a point such that $\tilde{U}_i(x;\gamma_{ij}) = \tilde{U}_j(x;\gamma_{ij})$ for all $i, j \in \{m, o, c\}$.

Finding the equilibrium of the flat pricing market is difficult because of the complex inter-play between $\boldsymbol{\alpha}$ and $\boldsymbol{p}$. We typically use a backward induction to solve the sequential game, that is, for a given $\boldsymbol{p}$, we solve the user's problem to find the corresponding $\boldsymbol{\alpha}(\boldsymbol{p})$. Then, we optimize the first stage game to decide on the equilibrium price $\boldsymbol{p}^\star$ by solving $\max_{\boldsymbol{p}} R(\boldsymbol{p})$ from (14). However, our problem requires a Newton-type backward induction to numerically solve a complex fixed point problem because of the lack of a closed form.

Note that even for a given $\boldsymbol{p}$, computing $\boldsymbol{\alpha}$ by solving the problem **User** is difficult. This is because the net-utility of each user affects $\boldsymbol{\alpha}$ because of the problem **User**, and also because $\boldsymbol{\alpha}$ affects the net-utility owing to the achieved data rates' dependence on $\boldsymbol{\alpha}$. In order to explicitly represent this dependency, we denote $\boldsymbol{x}(\boldsymbol{\alpha}) = (x^k(\boldsymbol{\alpha}) : k \in \{M, O, C\})$ for a given $\boldsymbol{\alpha}$.

Theorem 3.1 enables us to compute the equilibrium efficiently (the proof is presented in Appendix). The basic idea is that we express revenue $R = R(\boldsymbol{\alpha})$ as a function $\boldsymbol{\alpha}$, not $\boldsymbol{p}$ by finding $\boldsymbol{p}$'s closed form w.r.t. $\boldsymbol{\alpha}$. Then, revenue $R(\boldsymbol{\alpha})$ can be simply maximized. Note that for a given $\boldsymbol{\alpha}$, there may exist multiple values of $\boldsymbol{p}$ that lead to the same subscription ratio $\boldsymbol{\alpha}$. The set of all such $\boldsymbol{p}$ is denoted by $\mathcal{P}(\boldsymbol{\alpha})$. Theorem 3.1 also states that it is sufficient to consider *one* $\boldsymbol{p} \in \mathcal{P}(\boldsymbol{\alpha})$ because all price vectors in $\mathcal{P}(\boldsymbol{\alpha})$ lead the same.

For ease of presentation of Theorem 3.1, we will use the index variables i, j, and k to distinctly refer to one of the *service types* m, o, and c.

Theorem 3.1: Let $T_l := \sum_{K\in\{M,O,C\}} (x^K)^\theta \pi_l^K$, $l \in \{m, o, c\}$. Consider the following set $\mathcal{A}$:

$$\mathcal{A} \triangleq \{\boldsymbol{\alpha} \mid T_i \le T_j \le T_k\}.$$

For any $\boldsymbol{\alpha} \in \mathcal{A}$, there exists a $\boldsymbol{p}' = (p'_m, p'_o, p'_c) \in \mathcal{P}(\boldsymbol{\alpha})$, such that

$$\begin{aligned} p'_i &= U_i(\boldsymbol{x}(\boldsymbol{\alpha});\gamma_i), \\ p'_j &= U_j(\boldsymbol{x}(\boldsymbol{\alpha});\gamma_{ij}) - U_i(\boldsymbol{x}(\boldsymbol{\alpha});\gamma_{ij}) + p'_i, \\ p'_k &= U_k(\boldsymbol{x}(\boldsymbol{\alpha});\gamma_{jk}) - U_j(\boldsymbol{x}(\boldsymbol{\alpha});\gamma_{jk}) + p'_j, \end{aligned} \quad (19)$$

where $\gamma_i = 1 - \alpha_i - \alpha_j - \alpha_k$, $\gamma_{ij} = 1 - \alpha_j - \alpha_k$, and $\gamma_{jk} = 1 - \alpha_k$. Moreover, for any $\boldsymbol{p} \in \mathcal{P}(\alpha)$, the provider's revenue is identical.

IV. Partial Volume Pricing Market

A. Market Model

We also consider a partial volume pricing market model. The market is slightly different from the flat pricing market in that the provider should decide on the volume-based price, p_v^M, when a user is served by a macro BS, and a user should also decide on the elastic data demand x^M. In this section, we only consider the case of the *open-to-femto* policy for the following reason. When $p_m = 0$, in the *open-to-all* policy, *mobile-only* users can use a free *open-femto* service, in which case the provider's revenue is significantly reduced because of free-riding.

The provider selects the optimal prices that maximize the following problem:

$$\textbf{Provider}: \quad \begin{array}{ll} \max_{p_v^M, p_o, p_c} & R \\ \text{s.t} & p_v^M, p_o, p_c \ge 0, \end{array} \quad (20)$$

Then, a user with type γ first determines the data demand for macro BSs $x^M(\gamma)$ by maximizing the corresponding surplus subject to the macro BS capacity constraint. He or She then selects a service type to maximize the net-utility.

$$\textbf{User}: \quad \begin{aligned} x^M(\gamma) &= \arg\max_x \gamma x^\theta - p_v^M x, \\ j^*(\gamma) &= \arg \max_{j\in\{m,o,c\}} \tilde{U}_j(\boldsymbol{x};\gamma). \end{aligned} \quad (21)$$

The total amount of traffic must be less than the capacity of the macro BS. Thus, every user can be served according to his or her entire demand $x^M(\gamma)$, when the following condition is satisfied.

$$T^M \triangleq N \int \pi_{j(\gamma)}^M x^M(\gamma) d\gamma \le C_M, \quad (22)$$

where T^M denotes the total macro BS traffic generated by users. When T^M exceeds C_M, as in flat pricing, we assume that the fair scheduler controls the serving rate. Thus, the service rate is suppressed by an upper bound x_{max}^M, where

$$T^M = N \int \pi_{j(\gamma)}^M \min\{x_{max}^M, x^M(\gamma)\} d\gamma = C_M. \quad (23)$$

Note that unlike flat pricing where some users exit from the market and subscribe to no services, every user selects one of the services in partial volume pricing. The revenue of the operator simply reads:

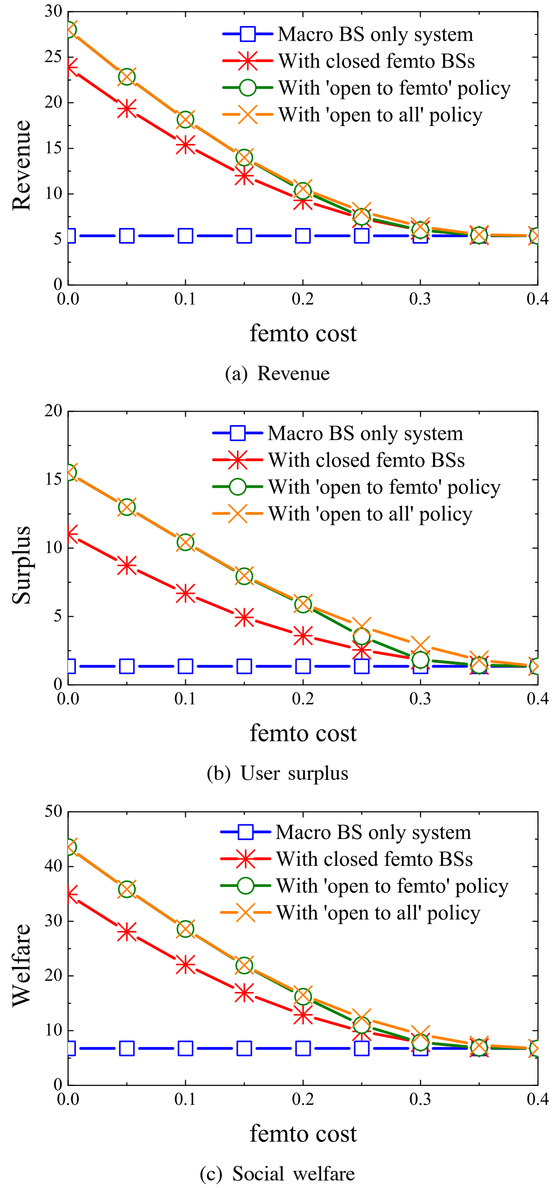
$$R = T^M p_v^M + N\{(p_o - c_f)\alpha_o + (p_c - c_f)\alpha_c\}. \quad (24)$$

B. Equilibrium

Computing the equilibrium in partial volume pricing is even harder than it is in flat pricing. This difficulty is caused by the hybrid structures of the two pricing schemes, where the volume pricing for macro BSs often causes the net-utility to be non-linear. When the net-utility graph has only one intersection point between any two lines, it is relatively easy to find the equilibrium point (as in the flat pricing case). However, this non-linear net-utility graph sometimes generates multiple intersections between any two net-utility curves, leading to difficulties in finding the relationship between $\boldsymbol{\alpha}$ and γ, which is the first step in computing the equilibrium.

For simplicity, we consider the case when $\bar{\gamma} = 1$. Similarly to flat pricing, we develop a theorem to compute the equilibrium. We again use the notations $x^k(\boldsymbol{\alpha})$, $k \in \{O, C\}$ and $x^M(\boldsymbol{\alpha};\gamma)$ to explicitly show the dependence of the data rates on $\boldsymbol{\alpha}$. We use $\boldsymbol{x}(\boldsymbol{\alpha})$ to denote the vector of $x^k(\boldsymbol{\alpha})$, and omit γ for notational simplicity, unless required.

In the description of Theorem 4.1, similarly to Theorem 3.1, we use the index variables i, j. When $x^O(\boldsymbol{\alpha}) > x^C(\boldsymbol{\alpha})$, $i =$ 'o', $j =$ 'c', otherwise, $i =$ 'c', $j =$ 'o'.

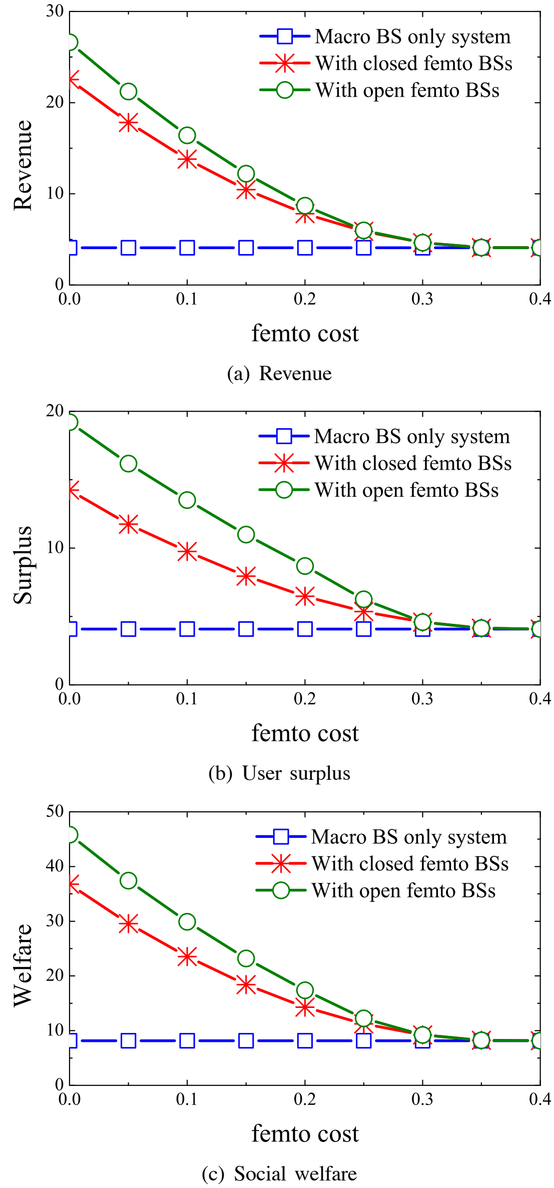


(a) Revenue

(b) User surplus

(c) Social welfare

Fig. 2. Flat pricing: value-added of the femto services ($N = 200$, $\beta = 0.0048$, $\theta = 0.5$, $\eta = 1$)



(a) Revenue

(b) User surplus

(c) Social welfare

Fig. 3. Partial volume pricing: value-added of the femto services ($N = 200$, $\beta = 0.0048$, $\theta = 0.5$, $\eta = 1$).

Theorem 4.1: We define $\mathcal{A}$

$$\begin{aligned} \mathcal{A} \ &\triangleq \ \{\boldsymbol{\alpha} \mid x^M(\boldsymbol{\alpha};\gamma) \\ &\leq \ \min\{x^O(\boldsymbol{\alpha}), x^C(\boldsymbol{\alpha})\}, \text{for all } \gamma \in (0,1]\}. \end{aligned} \quad (25)$$

Then, for all $\gamma \in (0, 1]$ and any given $\boldsymbol{\alpha} \in \mathcal{A}$,

(i) The γ_{mi} and γ_{ij} are unique and given by:

$$\gamma_{mi} = 1 - \alpha_i - \alpha_j, \quad \gamma_{ij} = 1 - \alpha_j. \quad (26)$$

(ii) The p_i and p_j are then expressed as a closed form of $\boldsymbol{\alpha}$ in the following manner:

$$\begin{aligned} p_i \ &= \ U_i(\boldsymbol{x};\gamma_{mi}) - U_m(\boldsymbol{x};\gamma_{mi}) + p_v^M x^M(\boldsymbol{\alpha};\gamma_{mi}), \\ p_j \ &= \ p_i + U_j(\boldsymbol{x};\gamma_{ij}) - U_i(\boldsymbol{x};\gamma_{ij}). \end{aligned} \quad (27)$$

(iii) $p_v^M = \infty$ maximizes the provider's revenue if

$$\frac{\theta}{2-\theta} \frac{(1-\pi_i^M)(\gamma_{mi})^{\frac{2-\theta}{1-\theta}} + \pi_i^M}{(1-\pi_i^M)(\gamma_{mi})^{\frac{1}{1-\theta}}(1-\gamma_{mi})} < 1. \quad (28)$$

Otherwise, the following p_v^M maximizes the revenue:

$$p_v^M = \theta \left(\left(\frac{1-\theta}{2-\theta} \right) \left((1-\pi_i^M)\gamma_{mi}^{\frac{2-\theta}{1-\theta}} + \pi_i^M \right) \left(\frac{N}{C_M} \right) \right)^{1-\theta}. \quad (29)$$

The proof is presented in the Appendix. Note that $\mathcal{A}$ contains all $\boldsymbol{\alpha}$ where femtocells give better throughput to users than that of macrocells. Therefore, $\mathcal{A}$ includes all cases where femtocells generate additional revenue to the operator.

Theorem 4.1(i) states that a relationship between the subscription ratio and the type γ can be simply characterized. Theorem 4.1(ii) represents p_o and p_c as simple functions of $\boldsymbol{\alpha}$. In Theorem 4.1(iii), the equations (28) and (29) are the functions of $\boldsymbol{\alpha}$, for example, γ_{mi} and π_i^M are determined for a given $\boldsymbol{\alpha}$. Thus, a simple optimization can be used to compute the equilibrium as in Theorem 3.1.

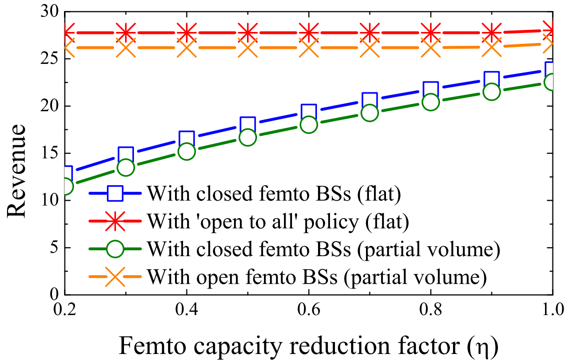


Fig. 4. Impact of Femto capacity reduction factor on the Revenue ($N = 200$, $\beta = 0.0048$, $\theta = 0.5$, $c_f = 0$)

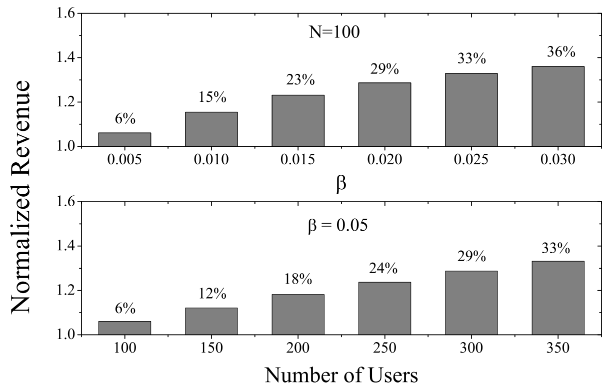


Fig. 5. Normalized revenue of open-femto BSs by that of just closed-femto BSs ($\theta = 0.5$, $\eta = 1$, $c_f = 0$). The percentages on the top of bars indicates the revenue increasing by open femto BSs.

In Theorem 4.1(iii), the left-hand side of the condition (28) is more likely to be satisfied when there are more femto users, because γ_{mi} declines for such a case. Thus, it means that with many femto users, p_v^M should be large to increase revenue, because with small p_v^M, more users tend to subscribe to the *mobile-only* service. In such a case, the provider will decrease p_o and p_c to attract more femto users, resulting in an overall decrease in revenue. Note that for a small number of femto users (i.e., (28) is violated), the provider gets more revenue with smaller p_v^M, wherein the data demand for macro BSs will grow. Thus, in order to maximize revenue, p_v^M is decreased to (29) until the data demand at macro BSs reaches its capacity. Note that even when p_v^M is very large, where users decrease the traffic demand for macro BSs, γ_{mi} can be some positive value.

V. Numerical Results

A. Setup

We now provide numerical results, where in most cases, we plot the provider's revenue, user surplus, social welfare, and user subscription ratio for different values of femto costs, pricing schemes, and users' price-sensitivities. We tested different values and observed similar trends to those presented in this section.

We consider a cellular network with N users/cells, where N is tested ranging from 100 to 350. Both C_M and C_F are set equal to 1. Note that the actual numbers of C_M and C_F are not critical, because revenue, user surplus, and social welfare just scale with those numbers; our main interest lies in investigating the relative ratios and changes of the metrics. The C_F can vary according to the transmission rate of the backbone network or the transmission power level of the femto BS. However, the ratio of C_M to C_F seems realistic, because the power level of femto BSs is set to give the same SINR to users on the boundary between femtocells and macrocells [18]. The probability of users being inside is set to be 0.4 [7]. The value β, the coverage of femto BSs (normalized by that of a macro BS) is tested over $[0.0048, 0.03]$, where we use 0.0048 unless explicitly mentioned. This value is obtained for macro and femto cells with radiuses of 500 m and 20 m, respectively, and macrocells exploit a three-sector topology. The value of maximum user type, $\bar{\gamma}$ is set to 1. For all simulations, unless explicitly mentioned, the price sensitivity is chosen as 0.5, which is the median of the interval $[0, 1]$. We vary the femto capacity reduction factor η in the interval $[0.2, 1]$, where we use $\eta = 1$ unless explicitly mentioned.

B. Value-added of the Open-Femto Service

Figs. 2 and 3 show the impact of *open-femto* services on the revenue and user surplus, when $\eta = 1$. We compare three different cases: *1)* no femto, *2)* only with closed-femto BSs, and *3)* with both closed and open femto BSs. We first observe that both revenue and user surplus increase with the introduction of *closed-femto* services for all pricing schemes, as also reported in [15]. From these results, we can compute social welfare, which also increases thanks to the *closed-femto* service, because social welfare is the sum of revenue and user surplus. The introduction of *open-femto* services, further increases revenue and user surplus because opening femtocells increases the total capacity of the system [8].

Fig. 4 shows the impact of interference between femto BSs and the macro BS. The x-axis and y-axis correspond to the interference factor and the revenue, respectively. The revenue with the open femto services is constant while that with the closed femto service decreases as η decreases. The differences between the open femto and the closed femto increase even more as η decreases, which shows the superiority of the open femto policy. As more macro users interfere femto BSs, the value of closed-femto BSs decreases linearly.

The value-added of open femtocells changes depending on the coverage β of open femto BSs and the number of users N. In Fig. 5, the normalized revenue of open-femto BSs by that of just closed-femto BSs is monotonically increasing, as β and N increase. This is because more offloading can be achieved by open-femto BSs than closed-femto BSs.

The value-added of open femto services decreases with increasing femto costs. This reduction is because more users select the *macro-only* service with high femto costs, as also

[7]According to [1], the probability of being at home is 40% and being at work is 30%, roughly. As our focus is on the individual user's acceptance of femto BSs, we use the probability of being home as that of being inside.

[8]It is called *positive externality* in economics.

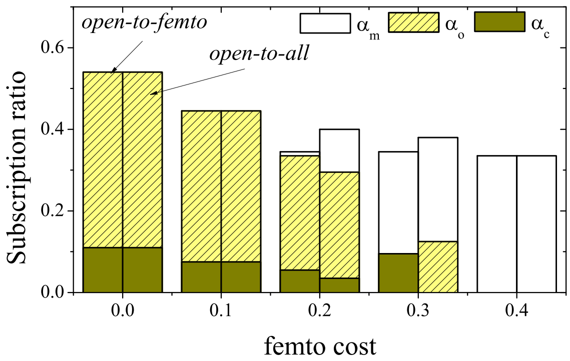


Fig. 6. Users' subscription ratios in flat pricing ($N = 200$, $\beta = 0.0048$, $\theta = 0.5$, $\eta = 1$)

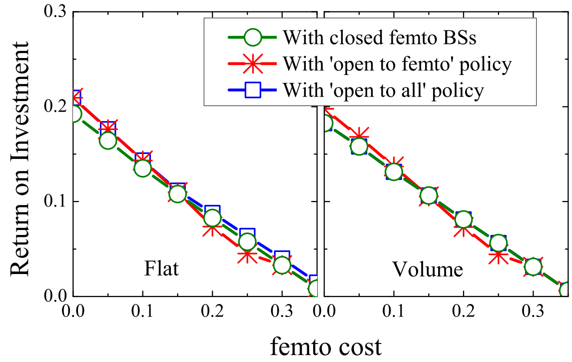


Fig. 7. Return on Investment (ROI) of femto BSs ($N = 200$, $\beta = 0.0048$, $\theta = 0.5$, $\eta = 1$)

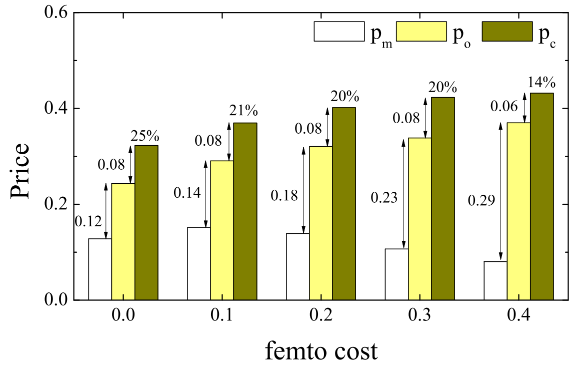


Fig. 8. Prices in open-to-all policy in flat pricing. The percentages on top of the bar graphs represent the subsidy, calculated by $\frac{p_c - p_o}{p_c}$. ($N = 200$, $\beta = 0.0048$, $\theta = 0.5$, $\eta = 1$)

shown in Fig. 6. In our environments, for femto costs higher than 0.4, no value-added is observed. Note that the cost 0.4 is very high in that 0.13 is the price that maximizes the revenue without femto BSs, that is, the cost 0.4 is approximately three times larger than the price of the macro-only service. Theorem 5.1 supports the observations above, stating that when the femto-cost is not significantly high, the open-femto service generates higher revenue than the macro-only service under flat pricing does. For a given femto cost c, $R_M(c)$, $R_A(c)$ and $R_F(c)$, denote the maximum revenues with no *open-femto* service, *open-femto* with *open-to-all*, and *open-femto* with *open-to-femto,* respectively.

Theorem 5.1: Under flat pricing, there exist numbers $\bar{c}_A$ and $\bar{c}_F$ such that $R_A(c) \geq R_M(c)$ for any $0 \leq c \leq \bar{c}_A$ and $R_F(c) \geq R_M(c)$, for any $0 \leq c \leq \bar{c}_F$. Moreover, $\bar{c}_A > \bar{c}_F$.

The proof and detailed expressions of $\bar{c}_A$ and $\bar{c}_F$, are presented in the Appendix. In order to offer practical insight, we numerically computed $\bar{c}_A$ and $\bar{c}_F$ for $\theta = 0.5$: $\bar{c}_A = 0.38$ and $\bar{c}_F = 0.36$. Thus, in flat pricing, the *open-to-all* policy is more economically robust to the femto cost, and, in our environments, it is verified that the bounds in Theorem 5.1 are a good match for those from the numerical computation.

Fig. 7 shows the Return on Investment (RoI) of femto BSs over femto costs for both pricing schemes. RoI intuitively refers to the increase in revenue compared to the invested capital. We assume that the investment is proportional to the number of deployed femto BSs. Thus, we define RoI as $(R - R_{\text{macro}})/(\alpha_o + \alpha_c)N$, where R and R_{macro} denote operator revenue and the revenue only with macro BSs, respectively. In practice, RoI is a useful metric for making decisions on investment, which gives an insight into how long it will take to recover the investment to the femtocell services [26]. For both pricing schemes, the RoIs of the three deployment scenarios are similar (see Fig. 7), whereas, in the revenue graph, the revenue of "with closed femto BS only" is significantly smaller than other "with 'open to femto' policy". This is because open femto BSs are promoted by a subsidy so that the number of femto BSs can increase, although revenue increases more with open femto BSs.

C. Impact of Femto Cost on User Behavior, Price, and Subsidy

The operator may want to provide a subsidy to motivate femto users to open their femto BSs since the utility of *closed-femto* service always exceeds that of *open-femto*. We define subsidy as the fraction $\frac{p_c - p_o}{p_c}$. This definition is adopted to reflect the fact that since open-femto users induce positive externalities, operators discount subscription fees for the femto service. Then, interesting questions include (i) how much subsidy is necessary, and (ii) how may users will subscribe to each service for the given subsidy.

Fig. 6 shows how the user behavior changes when femto costs or femto open policies vary under flat pricing. We observe that when the femto cost is low ($c_f \leq 0.2$), the majority of users join the *open-femto* or *closed-femto* services, whereas the subscription ratio decreases significantly as femto cost increases ($c_f > 0.2$). As shown in Fig 8, the provider whose objective is revenue maximization selects low prices for the *mobile-only* service for high femto cost, in which case, for the provider it is hard to attract more users subscribing to femto services.

Our numerical study suggests that the subsidy ranges between 10% and 20%, as shown in Fig. 8. We also observe that the provider may still start an open-femto business despite $(p_o - p_m) < c_f$, which implies that it should pay more money to install and maintain a femtocell than the increased price from introducing femtocells. This is illustrated in Fig. 8 for the femto cost > 0.2 and the *open-to-all* policy. Once again, the reason for this is the positive externalities of open femto BSs. Under the regime of non-negligible portion of *mobile-only*

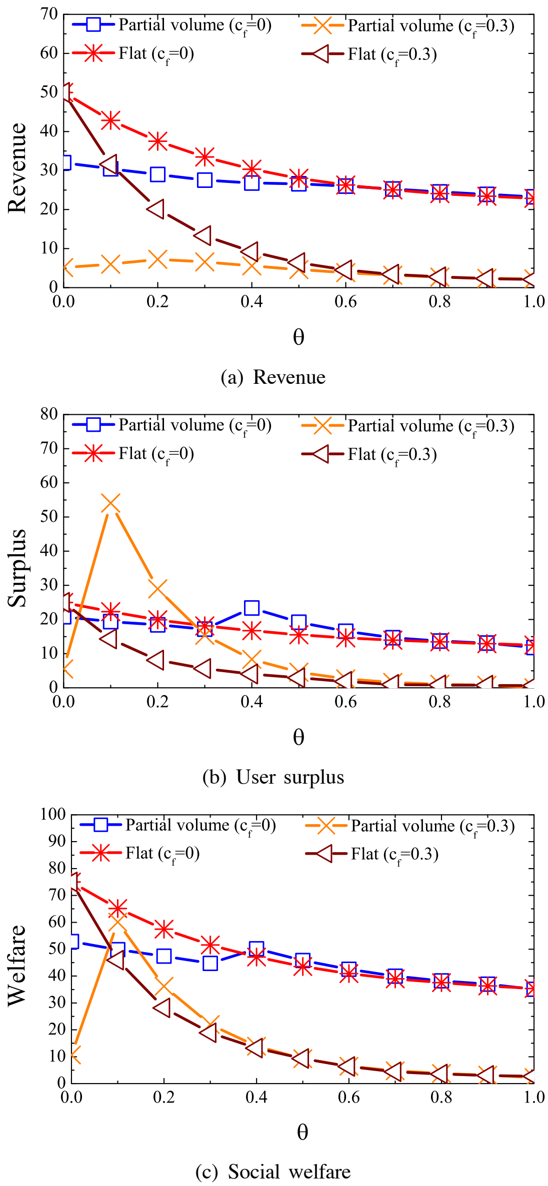


Fig. 9. Impact of pricing schemes ($N = 200$, $\beta = 0.0048$, $\eta = 1$)

users, the provider can increase the price p_m and thus increase the revenue earned from the *mobile-only* users. Despite a sufficient subsidy, the operator sustains high revenues, because more *open-femto* users lead both femto users and *mobile-only* users to increase their utilities and thus a high price is acceptable to users.

When interference factor η is considered, e.g., $\eta < 1$, *open-femto* users start to appear without any subsidy, since the larger capacity of open femto BSs provides enough incentive to open BSs. For example, in our simulation, when $\eta < 0.7$, no *closed-femto* users exist without subsidy. Thus, we can conclude that restrictive use of resource for closed femto BSs due to interference can be an enough incentive to open.

D. Open-to-all vs. Open-to-femto Policies

Fig. 2 shows that the plots for the two polices are close for all values of femto costs, where a small economic gain is observed in the *open-to-all* policy over the cost range $[0.2, 0.4]$. When the femto cost is less than 0.2, it is trivial that there is no difference between *open-to-all* and *open-to-femto* because users do not subscribe to the *mobile-only* service as shown in Fig. 6. On the other hand, over the cost range $[0.2, 0.4]$, *mobile-only* and femto service users can coexist and thereby, *mobile-only* users influence the economic aspects of the system. Over this cost range, the *open-to-all* policy produces only positive effects on user surplus because open femto BSs offload more data. However, the gain on revenue is minor because under *open-to-all* the price for femto services should be discounted due to the loss on their utility by the *mobile-only* users' access, whereas the revenue earned from *mobile-only* users increases.

However, the impacts of these open policies on user behavior is noteworthy. When c_f is low, user behavior is almost identical regardless of the policy. Differences exist only when the femto cost is high. Under the *open-to-femto* policy, users that subscribe to the femto service decide not to share their femto services, whereas under the *open-to-all* policy, users choose the *open-femto* service instead of the *closed-femto* service. Users seem to have higher incentives to share their femto BSs under the *open-to-all* policy than they do under the *open-to-femto* policy. Because sharing helps macro users as well as femto users, the operator can provide enough subsidy to persuade users to share their femto BSs.

Decrease in subsidy when $c_f = 0.4$ does not have a strong impact on the overall analysis, because for such a high cost, the users of femto services are extremely small or even disappear. We model the total femto operational costs as being linearly proportional to the number of femto users, as seen in (10). However, this model may not reflect certain practical cases. Thus, the results for high femto costs, for example, $c_f > 0.3$ may not deserve much attention.

E. Flat vs. Partial Volume

We now study the impact of pricing schemes. As shown in Fig. 9, we observe that in flat pricing the revenue is no less than that with partial volume pricing over most values of elasticity parameter, θ. In particular, smaller θ results in a significant gap in revenue between two pricing schemes. We interpret this result as follows:

In flat pricing, it is widely known that the users with higher willingness to pay, that is, γ, tend to dominate the network resources [1], [27]. This "negative externality" (i.e., congestion) due to users' heterogeneity in terms of willingness to pay in flat pricing can be alleviated in various ways which includes QoS-provisioning mechanism, i.e., imposing the maximum rate on the users with high demands or guaranteeing the minimum rate to the users with small demands. Volume pricing can clearly be another solution that lets the users with higher demand pay more. Adding QoS control to flat pricing often leads to larger revenue than volume pricing [28]. In our model, scheduling across users in a cell is assumed to be fair, and thus each user is served with a similar rate differently from the actual demand, which behaves like a QoS-control mechanism mentioned above. Note that this assumption is not significantly impractical. For example, in Korea, operators still adopt flat pricing (i.e., unlimited plan), but, with a QoS control which constrains users' maximum usage per day.

The revenue difference between two pricing schemes becomes larger with smaller θ. Note that θ is an elasticity parameter, i.e., as θ goes to 1, the traffic demand significantly varies as the user type γ, whereas, as θ goes to 0, the traffic demand is insensitive to the user type γ, and thus whether users are served or not itself affects the provider's revenue, not the traffic demand. From a simple calculation, the traffic demand over a macro BS under volume pricing, $x^M(\gamma)$, for a given user type γ, is given by $x^M(\gamma) = (\frac{\gamma\theta}{p})^{1/(1-\theta)}$. Thus, for low θ, users tend to transmit a (relatively) small volume of data with volume pricing, even if the price p_v^M is low. However, under flat pricing, the maximum price which guarantees a positive net-utility is γx^θ (due to the condition of $\gamma x^\theta - p \geq 0$) and γx^θ increases as θ decreases (due to $x < 1$). Thus, users are willing to subscribe to the service even with high price, which leads to higher revenue in flat pricing.

In regard to user surplus, we observe that a sharp increase of surplus for some specific value of θ: $\theta = 0.4$ for $c_f = 0$ and $\theta = 0.1$ for $c_f = 0.3$ in Fig. 9. We resort to Theorem 4.1 to interpret this. For small values of θ, the LHS of the condition (28) is likely to be met, where the provider chooses $p_v^M = \infty$ to maximize the revenue. In that case, no users generate data traffic at macro BSs, which connects large decrease in surplus. As θ increases, the condition (28) starts to be unsatisfied, then users accordingly start to use macro BSs with the price of (29) and experience the increase in surplus. For very high θ (very low price-sensitivity), the provider can attract the users with the increased p_v^M, again reduces the user surplus.

VI. Concluding Remarks

In this paper, we developed an analytical framework in order to study the economic aspects of the femtocell services on monopoly market. In particular, we focus on investigating the economic benefits of openness of femtocell networks based on the decision of users when the investment on femtocell networks are already taken. Under the developed model we drew the following conclusions:

1) With an enhanced network capacity driven by open-femto BSs, the open-femto service also benefits users as well as providers because users can enjoy cheaper services (i.e., subsidy) with better quality. Social welfare also increases in this scenario.
2) When *open-femto* services are offered, blocking the access of mobile-only users to femto BSs does not significantly influence revenue, user surplus or social welfare. Providers can choose either option depending on their preferences without the loss of economic benefit.
3) When users are price-sensitive, providers can achieve higher revenues with flat pricing than they can with partial volume pricing. However, the difference becomes negligible for users that have low price-sensitivity.

We comment that our conclusion 2) is based on the assumption that both femto/macro BS's capacity is not differentiated between two open-femto policies. In practice, in open-to-all policy the actual system capacity may be larger than that in open-to-femto policy due to interference. Thus, there may exist a gap in terms of revenue between two policies, which is left for future work. Future work also includes the study of the relation between the user subscription ratio $\boldsymbol{\alpha}$ and the femto capacity reduction factor η, where the value of η may differ for the closed femto BSs and the open femto BSs, under *open-to-femto* policy. Extending this work, it is interesting to see the followings: (i) the study of the cases when there exist multiple providers and the impact of their cooperation and competition [9] and (ii) comparison with other competitive technologies (e.g., picocell and WiFi).

Appendix

Proof of Theorem 3.1. For each of γ_i, γ_{ij}, and γ_{jk}, we find that $\tilde{U}_i(\boldsymbol{x};\gamma_i) = 0$, $\tilde{U}_i(\boldsymbol{x};\gamma_{ij}) = \tilde{U}_j(\boldsymbol{x};\gamma_{ij})$, and $\tilde{U}_j(\boldsymbol{x};\gamma_{jk}) = \tilde{U}_k(\boldsymbol{x};\gamma_{jk})$. Thus, at $\boldsymbol{p}'$, the equilibrium exists with $\boldsymbol{\alpha}$ since $T_i \leq T_j \leq T_k$. For the given $\boldsymbol{\alpha}$, p_l, where $l \in \{m, o, c\}$, can take other values only when $\alpha_l = 0$. ■

Proof of Theorem 4.1. At the p_i and p_j, $\tilde{U}_i(\boldsymbol{x};\gamma_{ij}) = \tilde{U}_j(\boldsymbol{x};\gamma_{ij})$. Moreover, the relationship $\tilde{U}_i(\boldsymbol{x};\gamma) > \tilde{U}_j(\boldsymbol{x};\gamma)$ holds when $\gamma < \gamma_{ij}$, and the relationship $\tilde{U}_i(\boldsymbol{x};\gamma) < \tilde{U}_j(\boldsymbol{x};\gamma)$ holds when $\gamma > \gamma_{ij}$. Thus, $\tilde{U}_i(\boldsymbol{x};\gamma)$ and $\tilde{U}_j(\boldsymbol{x};\gamma)$ intersect uniquely at γ_{ij}.

The utility difference between service type i and *mobile-only* service is given by:

$$\tilde{U}_i(\boldsymbol{x};\gamma) - \tilde{U}_m(\boldsymbol{x};\gamma) = \gamma \sum_{K\in\{O,C\}} (x^K)^\theta \pi_i^K - (1-\pi_i^M)\big(\gamma(x^M(\gamma))^\theta - p_v^M x^M(\gamma)\big) - p_i. \quad (30)$$

For $\gamma_b < \gamma_a$, we have:

$$\begin{aligned}
&\tilde{U}_i(\boldsymbol{x};\gamma_a) - \tilde{U}_m(\boldsymbol{x};\gamma_a) + p_i && (31)\\
&= \gamma_a \sum_{K\in\{O,C\}} (x^K)^\theta \pi_i^K - (1-\pi_i^M)\big(\gamma_a(x^M(\gamma_a))^\theta - p_v^M x^M(\gamma_a)\big) && (32)\\
&\geq \gamma_b \sum_{K\in\{O,C\}} (x^K)^\theta \pi_i^K - (1-\pi_i^M)\big(\gamma_b(x^M(\gamma_a))^\theta - p_v^M x^M(\gamma_a)\big) && (33)\\
&\geq \gamma_b \sum_{K\in\{O,C\}} (x^K)^\theta \pi_i^K - (1-\pi_i^M)\big(\gamma_b(x^M(\gamma_b))^\theta - p_v^M x^M(\gamma_b)\big) && (34)\\
&= \tilde{U}_o(\boldsymbol{x};\gamma_b) - \tilde{U}_m(\boldsymbol{x};\gamma_b) + p_o, && (35)
\end{aligned}$$

where Eq. (33) is obtained from the condition $\min\{x^O, x^C\} \geq x^M(\gamma_a)$ and Eq. (34) is obtained from the fact that $x^M(\gamma_b)$ maximizes $\gamma_b x^\theta - p_m^V x$. Thus, $\tilde{U}_i(\boldsymbol{x};\gamma) - \tilde{U}_m(\boldsymbol{x};\gamma)$ is increasing in γ, and γ_{mi} and γ_{ij} are uniquely determined with $\gamma_{ij} = 1 - \alpha_j$ and $\gamma_{mi} = \gamma_{ij} - \alpha_i$.

Assume that the capacity for macro BSs is not constrained. Remarking that:

$$\frac{dU_l(\boldsymbol{x};\gamma)}{dx^M} = \pi_l^M\big(\theta\gamma(x^M)^{\theta-1} - p_v^M\big), \qquad l \in \{m, o, c\} \quad (36)$$

[9] The duopoly case is an ongoing work [29].

the service rate of macro BSs for users with type γ is $x^M(\gamma) = (\frac{\theta\gamma}{p_v^M})^{\frac{1}{1-\theta}}$. We also find that:

$$\frac{dR}{dp_v^M} = N\theta^{\frac{1}{1-\theta}}(p_v^M)^{\frac{1-2\theta}{1-\theta}}\Big\{(1-\pi_i^M)(\gamma_{mi})^{\frac{1}{1-\theta}}(1-\gamma_{mi}) - \frac{\theta}{2-\theta}((1-\pi_i^M)(\gamma_{mi})^{\frac{2-\theta}{1-\theta}} + \pi_i^M)\Big\}. \quad (37)$$

Thus, when the condition (28) is met, the revenue increases with p_v^M. Otherwise, owing to the capacity constraint, the revenue is maximized when p_v^M satisfies following:

$$\begin{aligned} C_M &= \int \pi_{j(\gamma)}^M(\frac{\theta\gamma}{p_v^M})^{\frac{1}{1-\theta}} d\gamma \\ &= N\Big(\frac{1-\theta}{2-\theta}\Big)\Big((1-\pi_i^M)(\gamma_{mi})^{\frac{2-\theta}{1-\theta}} + \pi_i^M\Big)\Big(\frac{\theta}{p_v^M}\Big)^{\frac{1}{1-\theta}}. \end{aligned}$$

This concludes the proof. ■

Proof of Theorem 5.1. We prove the theorem by finding the conditions on the femto costs for *open-to-all* and *open-to-femto*, such that revenue when there are only *mobile-only* users increases along with the change from a mobile-only user to a femto user.

Initially, let α be the subscription ratio for the system with only *mobile-only* users. Subsequently, by jointly solving (13) and (15), R in (14) is given by: $R = (1-\alpha)(C_M/(\alpha N))^\theta \alpha N$, where we get:

$$\begin{aligned} \frac{dR}{d\alpha} &= N\Big(\frac{C_M}{N}\Big)^\theta \alpha^{-\theta}(1-\theta-(2-\theta)\alpha), \quad (38) \\ \frac{d^2R}{d\alpha^2} &= -N\Big(\frac{C_M}{N}\Big)^\theta (1-\theta)(2+\theta(1-\alpha)\alpha^{-1}). \quad (39) \end{aligned}$$

Since Eq. (39) is non-positive for $\alpha > 0,\ \theta \in [0,1]$, R is concave in α. Thus, R is maximized when $\alpha^* := \frac{1-\theta}{2-\theta}$. Let γ_m^b be $1-\alpha^*$. Then, when no femto BS exists, according to Theorem 3.1, the maximum revenue is $U_m(\boldsymbol{x}^{(0)};\gamma_m^b)\alpha^* N$, where $\boldsymbol{x}^{(0)}$ is a traffic rate vector.

Let us assume that, under the *open-to-all* policy, a user, whose type value is $\bar{\gamma}$, changes his or her service to *open-femto*. Then, there are $\alpha^* N - 1$ *mobile-only* users and an *open-femto* user. From Theorem 3.1, the prices for *mobile-only* and *open-femto* are defined as

$$\begin{aligned} p_m^{(1)} &:= U_m(\boldsymbol{x}^{(1)};\gamma_m^b), \quad (40) \\ p_o^{(1)} &:= U_o(\boldsymbol{x}^{(1)};1) - U_m(\boldsymbol{x}^{(1)};1) + U_m(\boldsymbol{x}^{(1)};\gamma_m^b) \quad (41) \end{aligned}$$

where $\boldsymbol{x}^{(1)}$ is a traffic rate vector when there are $\alpha^* N$ subscribers and they are *mobile-only* users except one subscribing to the *open-femto* service. Thus, the revenue increment by introducing an *open-femto* user is computed as

$$R^* = (U_m(\boldsymbol{x}^{(1)};\gamma_m^b) - U_m(\boldsymbol{x}^{(0)};\gamma_m^b))\alpha^* N + (U_o(\boldsymbol{x}^{(1)};1) - U_m(\boldsymbol{x}^{(1)};1)) - c. \quad (42)$$

If R^* is positive, the provider can gain more revenue with an open femto BS. Let the bound $\bar{c}_O$ denote the maximum femto cost that guarantees more profit with an open femto BS. Then the bound $\bar{c}_O$ is represented as

$$\bar{c}_O = ((1-\alpha^*)\alpha^* N\delta_o\beta + \delta_i)(\frac{C_O}{1+\delta_i+\delta_o\beta\alpha^* N})^\theta - (1-\alpha^*)\alpha^* N(\frac{C_M}{1+\alpha^* N})^\theta + ((1-\alpha^*)\alpha^* N(1-\delta_o\beta) - \delta_i) \times (\frac{C_M}{1+\delta_i(\alpha^* N-1)+\delta_o(1-\beta)\alpha^* N})^\theta. \quad (43)$$

Similarly, we can also compute the bound $\bar{c}_C$ which is the maximum femto cost where a provider gains revenue by introducing one *closed-femto* user.

$$\bar{c}_C = \delta_i C_C^\theta + ((1-\alpha^*)\alpha^* N - \delta_i)(\frac{C_M}{1+\alpha^* N - \delta_i})^\theta - (1-\alpha^*)\alpha^* N(\frac{C_M}{1+\alpha^* N})^\theta. \quad (44)$$

Thus, under the *open-to-all* policy, $\bar{c}_A = \max(\bar{c}_O, \bar{c}_C)$. Moreover, under the *open-to-femto* policy, $\bar{c}_F = \bar{c}_C$, because only one *open-femto* user has the same characteristic as only one *closed-femto* user. Finally, we can conclude that $\bar{c}_A \geq \bar{c}_F$. This concludes the proof. ■

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