Supplement Material: On Maximizing Diffusion Speed over Social Networks with Strategic Users

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A. *Example in Section III-C*

In what follows, we provide an example mentioned in Section III-C, which shows that $\Gamma^*(\cdot)$ is neither supermodular nor submodular.

Consider a graph G consisting of two disconnected complete graphs G_1 and G_2, both of which has n nodes. Then, it is easy to show that for any seed set C in G_1, $\Gamma^*(G_1, C) = 0$ (we explicitly express the parameter G_1 if needed, but omit it for the graph G for notational simplicity), when $|C| \geq \frac{1-h}{2}n$, similarly applied to G_2, also. We first note that $\Gamma^*(G_1, \emptyset) = \Gamma^*(G_2, \emptyset)$, which corresponds to the diffusion exponent without any seeding. Then from symmetry and disconnectedness of G_1 and G_2, we observe that

$$\Gamma^*(\emptyset) = \Gamma^*(G_1, \emptyset) = \Gamma^*(G_2, \emptyset).$$

We now consider a seed set C_1 and C_2 in G_1, G_2, respectively, where $|C_1| = |C_2| = \frac{1-h}{2}n + 1$ (thus the diffusion exponent of both subgraphs is 0). Then, we have:

$$\Gamma^*(C_2) - \Gamma^*(\emptyset) > \Gamma^*(C_1 \cup C_2) - \Gamma^*(C_1),$$

since in LHS $\Gamma^*(C_2) = \Gamma^*(\emptyset) = \Gamma^*(G_1, \emptyset)$, and in RHS $\Gamma^*(C_1 \cup C_2) = 0$ and $\Gamma^*(C_1) = \Gamma^*(G_2, \emptyset) > 0$. This disproves supermodularity of $\Gamma^*(\cdot)$.

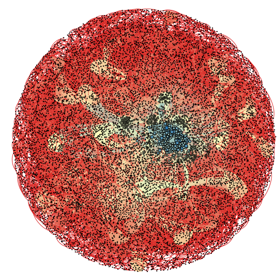
Also, to disprove submodularity of $\Gamma^*(\cdot)$, we additionally consider two seed sets C_1', C_2' in G_1, G_2, respectively, such that $C_l \cap C_l' = \emptyset$ with $|C_l| = |C_l'|$ for $l = 1, 2$ (note that we can do this for sufficiently large n since $\frac{1-h}{2} < \frac{1}{2}$). Then, we have:

$$\Gamma^*(C_1' \cup C_2') - \Gamma^*(\emptyset) < \Gamma^*(C_1' \cup C_2' \cup C_1 \cup C_2) - \Gamma^*(C_1 \cup C_2),$$

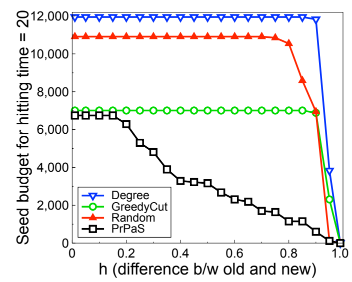
since, in the above, every term except $\Gamma^*(\emptyset)$ (which is positive) is 0. This also disproves submodularity of $\Gamma^*(\cdot)$.

B. *Additional Experiment*

We also use topology data set extracted from the collaboration network among high energy physicists in Arxiv. The data set is originally obtained in [1] and it forms an undirected graph where each node corresponds to an Arxiv account in and an edge between two accounts indicates that they have a joint work on high energy physics published in Arxiv. We name the graph HEParxiv, whose graphical presentations are given in Figure 1(a).



(a) HEParxiv consisting of $12,008$ users and $118,521$ edges and having average clustering coefficient 0.612 and degree distribution fit into power law distribution with exponent 1.44.



(b) Threshold(20) with varying h in HEParxiv.

Fig. 1. Blueprint of HEParxiv [1] and simulation result with HEParxiv.

We plot Threshold(20) with varying h in Figure 1(b). As you can see, PrPaS outperforms all other algorithms. We note that HEParxiv has clustering coefficient 0.612 and degree distribution fit into power law distribution with exponent 1.44. Although both the clustering coefficient and the power law exponent of HEParxiv are relatively high comparing to those of PPfacebook [2] and PLfacebook [3] and PLfacebook, the power law exponent of HEParxiv is particularly higher than others. Hence, as we observed with PLfacebook, one can also observe that GreedyCut outperforms Random due to the dominance of the power law exponent comparing to the clustering coefficient.

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References

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