Fair Greening for DSL Broadband Access*

Paschalis Tsiaflakis[1] Yung Yi[2] Mung Chiang[3] Marc Moonen[1]

[1] Electrical Engineering, Katholieke Universiteit Leuven, Belgium, {ptsiafla,moonen}@esat.kuleuven.be

[2] Electrical Engineering and Computer Science, KAIST, South Korea, yiyung@ee.kaist.ac.kr

[3] Electrical Engineering, Princeton University, USA, chiangm@princeton.edu

ABSTRACT

Given that broadband access networks are an integral part of the ICT infrastructure and that DSL is the most widely deployed broadband access technology, greening DSL has become important. Our recent work demonstrated a promising tradeoff between data rate performance and energy conservation. However, more greening still implies possibly lower data rate, and allocating this "price of greening" across interfering users needs to be fair. This paper proposes four formulations of fair greening in interference-limited networks, unifies them into one general representation, and develops a unified algorithm to solve them effectively. Simulations quantify the intuitions on fairness in greening DSL, as these four alternative approaches offer a range of choices between maintaining a high sum data rate and enforcing various definitions of fairness. Fairness of allocating the price of greening is also interesting in its own right.

1. INTRODUCTION

Greening the broadband access network is an essential part of the emerging trend towards Green Information and Communication Technology (Green ICT). The total number of broadband access lines worldwide is expected to grow from 393 mln in 2008 to 635 mln by the end of 2013. One of the most widely deployed broadband access technologies is digital subscriber line (DSL) that has a market share of over 65% and more than 200 million subscribers worldwide. DSL refers to a family of technologies that are capable of delivering broadband data rates over the copper wires of local telephone networks. One of the major obstacles to further performance improvement remains to be the electromagnetic interference, referred to as crosstalk, generated among different lines operating in the same cable bundle. This crosstalk is typically 10-20 dB larger than the background noise.

Dynamic spectrum management (DSM) has been developed as a key solution for tackling the crosstalk problem. The main idea of DSM is to prevent and/or remove crosstalk by spectrum and/or signal coordination of all users. In this paper we focus on spectrum coordination, also referred to as multi-carrier power control or spectrum balancing. The concept here is to jointly optimize the transmit power spectra of multiple interfering users so as to prevent the destructive impact caused by crosstalk interference. The last five years have seen a variety of powerful DSM algorithms to significantly increase the data rates of DSL networks. These algorithms range from fully autonomous [1, 12], to distributed [6, 8] and fully centralized DSM algorithms [2], where the main objective was all on data rate maximization without considering energy consumption.

As part of the push towards Green ICT (by regulatory organizations, e.g., ITU-T Study Group 15 and European Code of Conduct for broadband equipment [5]), reducing energy consumption of broadband equipment has now become an important consideration. In particular, the DSL Forum encourages international standards bodies to develop techniques for power reduction while preserving performance.

Energy-aware DSM is one of the most effective and readily deployable ways to enable *Green DSL*. It offers a general framework to manage transmit spectra so as to find a good trade-off between data rate performance and transmit energy consumption. In our recent work [9], it is shown that large power savings can be obtained with only minor degradations in data rate performance: a typical DSL deployment can have an $85-50$ tradeoff with 85% of the data rate maintained by spending only 50% of energy. In related work such as [11], the special case of total energy minimizing DSM is also studied.

However, what is left under-studied in green broadband access is the issue of *fairness*. Greening, either by imposing energy consumption constraints or by adding energy-aware terms in the objective function, will reduce performance metrics. This is the "price of greening". In an interference-limited physical layer such as DSL, how should this price of greening be allocated across the interfering users so that it satisfies some notion of fairness? This is the subject of the current paper.

Imposing fairness can change the shape of the tradeoff curve between performance (sum data rate) and greening (energy conservation). Furthermore, there is an interesting tradeoff between how fair a greening approach is and how much energy it can save. Among the four ways of fair greening proposed, one (method 3) can maintain the highest data rate and be greened systematically with fairness installed, while another (method 4) can provide completely

*This research work was carried out at the ESAT laboratory of Katholieke Universiteit Leuven, in the frame of K.U.Leuven Research Council CoE EF/05/006 Optimization in Engineering (OPTEC), Concerted Research Action GOA-MaNet, the Belgian Programme on Interuniversity Attraction Poles initiated by the Belgian Federal Science Policy Office IUAP P6/04 (DYSCO, 'Dynamical systems, control and optimization', 2007-2011), Research Project FWO nr.G.0235.07('Design and evaluation of DSL systems with common mode signal exploitation'), and is supported in part by Princeton Grand Challenge Grant, NSF ITR FAST Copper Project and a Google grant, and also by Korea Research Council of Fundamental Science & Technology

proportional fair greening. These discussions on fairness of allocating the price of greening among competing users may be of interest to other areas of Green ICT.

After reviewing the DSL physical layer interference model in Section 2, and our recently developed Green DSL framework in Section 3, we develop four notions of fairness in allocating the price of greening, resulting in four members of a new family of optimization problems in Section 4. We then show how a unified problem statement and solution algorithm can tackle this family of problems by leveraging and extending state-of-the-art DSM algorithms. This algorithm is tested using realistic DSL channel simulators, with key results summarized in Section 5.

2. DSL SYSTEM MODEL

We provide a brief introduction to DSL and refer the readers to [7] for details. Consider a DSL network with a set of $\mathcal{N} = \{1, \ldots, N\}$ transmitting modems (i.e., users, lines) and $\mathcal{K} = \{1, \ldots, K\}$ tones (i.e., frequency carriers). We make the standard assumption of perfect synchronization and discrete multitone modulation. Modems employ single-user encoding and decoding (treating interference as noise), resulting in the following expression for the achievable bit rate of modem n on tone k:

$$b_k^n(\mathbf{s}_k) \triangleq \log_2\left(1 + \frac{1}{\Gamma}\frac{|h_k^{n,n}|^2 s_k^n}{\sum_{m\neq n}|h_k^{n,m}|^2 s_k^m + \sigma_k^n}\right)\text{bits/Hz}, \quad (1)$$

where s_k^n denotes the transmit power of modem n on tone k, $\mathbf{s}_k = [s_k^1, \ldots, s_k^N]^T$ denotes the transmit power of all modems n on tone k, $[\mathbf{H}_k]_{n,m} = h_k^{n,m}$ is an $N \times N$ matrix containing the channel transfer functions from transmitter m to receiver n on tone k. The diagonal elements are the direct channels, the off-diagonal elements are the crosstalk channels. σ_k^n denotes the noise power on tone k in receiver n that contains thermal noise, alien crosstalk and radio frequency interference. Γ denotes the gap to capacity, which is a function of the desired bit error ratio (BER), the coding gain and noise margin [7].

The data rate of modem n, denoted by R^n, is the total sum of the data rate over tones considering the symbol rate, i.e., $R^n = f_s \sum_{k\in\mathcal{K}} b_k^n(\mathbf{s}_k)$, where f_s denotes the symbol rate. Let $\mathbf{R} = [R^1, \ldots, R^N]^T$. Denote by $P^n = \sum_{k\in\mathcal{K}} s_k^n$ the total transmit power of modem n. DSL standards impose the constraints on the transmit powers in terms of the total powers (2) as well as spectral masks (3), given by:

$$\mathbf{P} \preceq \mathbf{P}^{\text{tot}} \quad (2)$$

$$\mathbf{s}_k \in \mathcal{S}_k = \{\mathbf{s}_k \in \mathbb{R}^N : 0 \le s_k^n \le s_k^{n,\text{mask}}, n \in \mathcal{N}\}, \quad (3)$$

where '$\preceq$' denotes the element-wise inequality. The $\mathbf{P} = [P^1, \ldots, P^N]^T$ and $\mathbf{P}^{\text{tot}} = [P^{1,\text{tot}}, \ldots, P^{N,\text{tot}}]^T$ with $P^{n,\text{tot}}$ being the total power budget in modem n. The $s_k^{n,\text{mask}}$ corresponds to the spectral mask constraint for user n on tone k. Then, the set of all achievable combinations of data rates can be modelled by the achievable rate region $\mathcal{R}$:

$$\mathcal{R} = \Big\{(R^n : n \in \mathcal{N}) | R^n = f_s \sum_{k\in\mathcal{K}} b_k^n(\mathbf{s}_k), \text{s.t. (2) and (3)}\Big\}.$$

3. GREENING DSL

In our preliminary study [9], we proposed a unifying green DSL framework, formulated by the following problem:

$$\begin{aligned}\max_{\{s_k^n, k\in\mathcal{K}, n\in\mathcal{N}\}} \quad & \sum_{n\in\mathcal{N}} w_n R^n - \sum_{n\in\mathcal{N}} t_n P^n \\ \text{s.t.} \quad & P^n \le P^{n,\text{tot}} && n \in \mathcal{N}, \\ & 0 \le s_k^n \le s_k^{n,\text{mask}} && k \in \mathcal{K}, n \in \mathcal{N}, \\ & R^n \ge R^{n,\text{target}}, && n \in \mathcal{N}, \\ & \sum_{n\in\mathcal{N}} P^n \le \alpha \sum_{n\in\mathcal{N}} P^{n,\text{tot}}. && \end{aligned} \quad (4)$$

The above problem is a general physical layer resource allocation formulation that can model different operating points jointly in terms of data rates $(R^n : n \in \mathcal{N})$ and total transmit powers $(P^n : n \in \mathcal{N})$.

Choosing the proper values of $\{w_n, t_n, R^{n,\text{target}}, \alpha\}$ enables network operators to steer the efficient configuration between data rates and energy consumption. In particular, there are two alternative ways to green DSM to different degrees: (1) change the ratio between rate weight w and power weight t, and (2) change the energy consumption constraint α. Based on the generalized green DSL framework in (4), we showed that significant power savings (e.g., 50%) are feasible only with minor degradation in data rate (e.g., 15%) [9]. This desirable tradeoff is in part due to the details in the interference structure of DSL and the dependence of rate on the Signal-to-Interference-Ratio.

4. FAIRNESS IN GREENING DSL

However, what remains under-studied is the issue of fairness. The interfering users of a DSL network emit different amount of interference to other users, causing different degrees of difficulty in achieving a desirable "green vs. fast" tradeoff. How to allocate the price of greening, in terms of rate reduction, among the competing users now becomes important if green DSL is going to be standardized and deployed. In this and the next section, we study four different notions of fairness, which lead to four optimization problems that present different energy-performance tradeoff curves and yet can be solved in one unified way.

4.1 Fair greening formulations

There is no universally agreed notion of fairness, especially when applied to an emerging topic like Green ICT. Based on a range of reasonable views on what fairness is, we develop four formulations of Green DSL in this section, each parameterized by a "degree of greening" parameter $\beta \in [0, 1]$. Throughout this subsection, for a given greening formulation, we denote by $\{\boldsymbol{R}^o, \boldsymbol{P}^o\}$ the rate-power point somewhere on the border of the rate region $\mathcal{R}$ when $\beta = 1$, i.e., the point when no greening is applied.

(1) Energy-fair greening

In this approach, we target "perfect power fairness" by having weighted rate maximization as the objective, and proportionally reducing the available transmit power per modem:

$$\textbf{Greening 1:} \quad \begin{aligned}\max_{\{\mathbf{s}_k\in\mathcal{S}_k, k\in\mathcal{K}\}} \quad & \sum_{n\in\mathcal{N}} w_n R^n \\ \text{subject to} \quad & P^n \le \beta P^{n,\text{tot}},\ n \in \mathcal{N}. \end{aligned} \quad (5)$$

(2) Rate-fair greening

In contrast to the previous approach, this approach targets "perfect rate fairness" by having power minimization as the objective and by proportionally reducing the minimum

Table 1: Reduction of the unified formulation (9) into the four special cases

	Energy-fair	Rate-fair	Sum-rate-fair	Energy/Rate proportional
U	$\boldsymbol{w}^T\hat{\mathbf{R}}$	$-\mathbf{1}^T\hat{\mathbf{P}}$	$\boldsymbol{w}^T\hat{\mathbf{R}}+\delta\sum_n G^n(\hat{P}^n)$	$\boldsymbol{w}^T\hat{\mathbf{R}}$
$\boldsymbol{I}$	$\hat{\mathbf{P}}-\beta\mathbf{P}^{\text{tot}}$	$\begin{bmatrix}\hat{\mathbf{P}}-\mathbf{P}^{\text{tot}}\\ \beta\mathbf{R}^o-\hat{\mathbf{R}}\end{bmatrix}$	$\begin{bmatrix}\hat{\mathbf{P}}-\mathbf{P}^{\text{tot}}\\ \mathbf{1}^T\hat{\mathbf{P}}-\beta\mathbf{1}^T\mathbf{P}^{\text{tot}}\end{bmatrix}$	$\hat{\mathbf{P}}-\beta\mathbf{P}^{\text{tot}}$
$\boldsymbol{E}$	/	/	/	$\hat{\mathbf{R}}-\beta\mathbf{R}^o(\hat{\mathbf{P}}/\mathbf{P}^o)$
$\mathbf{f}$	$\boldsymbol{w}$	$[\lambda_{N+1},\ldots,\lambda_{2N}]^T$	$\boldsymbol{w}$	$\boldsymbol{w}+\boldsymbol{\mu}$
$\mathbf{h}$	$\boldsymbol{\lambda}$	$[\lambda_1,\ldots,\lambda_N]^T+1$	$\boldsymbol{t}+\epsilon(\mathbf{P}-\hat{\mathbf{P}})$	$\boldsymbol{\lambda}+\boldsymbol{\mu}\beta\mathbf{R}^o/\mathbf{P}^o$
$\hat{\mathbf{R}}^n$	/	/	/	/
$\hat{\mathbf{P}}^n$	/	/	$(G^n_{\hat{P}^n})^{-1}((\lambda_n+\lambda_{N+1}-t_n)/\delta)$	/

target data rate of users:

Greening 2:
$$\max_{\{\mathbf{s}_k\in\mathcal{S}_k,k\in\mathcal{K}\}} \quad -\sum_{n\in\mathcal{N}} P^n$$
$$\text{subject to} \quad P^n\le P^{n,\text{tot}}, \quad n\in\mathcal{N},$$
$$R^n\ge\beta R^{n,o}, \quad n\in\mathcal{N}. \quad (6)$$

(3) Sum-rate greening with fair regularization

Yet another alternative formulation is to start from the weighted sum rate formulation with a system total power constraint parameterized by β. However, this formulation does not impose any fairness consideration and indeed can return solutions that favor some users substantially. Therefore we augment the objective function with a fairness term G:

Greening 3:
$$\max_{\{\mathbf{s}_k\in\mathcal{S}_k,k\in\mathcal{K}\}} \quad \sum_{n\in\mathcal{N}} w_nR^n+\delta\sum_{n\in\mathcal{N}} G^n(P^n)$$
$$\text{subject to} \quad P^n\le P^{n,\text{tot}}, \quad n\in\mathcal{N}$$
$$\sum_{n\in\mathcal{N}} P^n\le\beta\sum_{n\in\mathcal{N}} P^{n,\text{tot}}, \quad (7)$$

where δ is a weighting factor tuned to emphasize the importance of greening fairness relative to the maximal weighted data rate performance. The function $G^n(P^n)$ may take different forms depending on the desired energy fairness. One possibility is α-fairness: $G^n(\cdot)=(\cdot)^{1-\alpha}/(1-\alpha)$, for $\alpha>0$, and $\log(\cdot)$, for $\alpha=1$, which includes max-min ($\alpha\to\infty$) and proportional fairness ($\alpha=1$) as special cases. Another possibility is to use second-moment as a measure of fairness: $G^n(.)=-(.)^2$. Therefore, we have the following three special cases:

- **Greening 3A:** (7) with $\delta=0$;
- **Greening 3B:** (7) with $\delta>0$ and $G^n(.)=-(.)^2$;
- **Greening 3C:** (7) with $\delta>0$ and $G^n(.)=\log(.)$.

(4) Energy/Rate proportional greening

In the fourth approach, the ratios of their data rate decrease and their total transmit power decrease are kept the same across the users: those that transmit at higher rate takes a proportionally larger share of the price of greening:

Greening 4:
$$\max_{\{\mathbf{s}_k\in\mathcal{S}_k,k\in\mathcal{K}\}} \quad \sum_{n\in\mathcal{N}} w_nR^n$$
$$\text{subject to} \quad P^n\le P^{n,\text{tot}}, \quad n\in\mathcal{N},$$
$$\frac{R^n}{R^{n,o}}/\frac{P^n}{P^{n,o}}=\beta, n\in\mathcal{N}. \quad (8)$$

This formulation requires no weight to be tuned as in (7). Note that $\beta=1$ corresponds to no greening and increasing β corresponds to greening. One can easily extend (8) with an extra bisection search to find the value of β that corresponds to a particular system power usage. This is because system power usage is monotone decreasing with increasing β.

4.2 Unified formulation and solution algorithm

Mathematically, the four fair greening methods in (5), (6), (7) and (8) can be shown as special cases of the generalized fair greening formulation:

$$\max_{\hat{\mathbf{R}},\hat{\mathbf{P}},\{\mathbf{s}_k\in\mathcal{S}_k,k\in\mathcal{K}\}} \quad U(\hat{\mathbf{R}},\hat{\mathbf{P}})$$
$$\text{subject to} \quad \boldsymbol{I}(\hat{\mathbf{R}},\hat{\mathbf{P}})\le 0, \quad \boldsymbol{E}(\hat{\mathbf{R}},\hat{\mathbf{P}})=0,$$
$$\hat{\mathbf{R}}=\mathbf{R}, \quad \mathbf{P}=\hat{\mathbf{P}}, \quad (9)$$

where the specific forms for $U(\hat{\mathbf{R}},\hat{\mathbf{P}})$, $I(\hat{\mathbf{R}},\hat{\mathbf{P}})$ and $E(\hat{\mathbf{R}},\hat{\mathbf{P}})$ corresponding to the four formulations, are given in Table 1.

Solving these problems is challenging since they are NP-hard non-convex problems. To tackle computational difficulty, we use Lagrange relaxation. The power of this approach lies in the fact that (asymptotic) strong duality holds despite the nonconvexity of the optimization problem [4] and that the overall problem can be easily decomposed into the many much simpler independent subproblems. We present the following three subproblems to be solved, based on the Lagrange relaxation of the original problem (9), that are reduced in dimension and are thus much simpler to solve. More details of this decomposition methodology can be found, e.g., in [1, 10].

P1. Find the optimal Lagrange multipliers $\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\nu},\boldsymbol{t}$, that enforce the constraints. This can be solved by standard subgradient based updates of the Lagrange multipliers where the subgradients are $\boldsymbol{I}(\hat{\mathbf{R}},\hat{\mathbf{P}})$ and $\boldsymbol{E}(\hat{\mathbf{R}},\hat{\mathbf{P}})$.

P2. Solve the convex unconstrained problem in $\hat{\mathbf{R}}$ and $\hat{\mathbf{P}}$:

$$\max_{\hat{\mathbf{R}},\hat{\mathbf{P}}} U(\hat{\mathbf{R}},\hat{\mathbf{P}})-\boldsymbol{\lambda}^T\boldsymbol{I}(\hat{\mathbf{R}},\hat{\mathbf{P}})-\boldsymbol{\mu}^T\boldsymbol{E}(\hat{\mathbf{R}},\hat{\mathbf{P}})$$
$$-\boldsymbol{\nu}^T\hat{\mathbf{R}}+\boldsymbol{t}^T\hat{\mathbf{P}} \quad (10)$$

Using first order necessary conditions, $\hat{\mathbf{R}}$ and $\hat{\mathbf{P}}$ are given by solving the following system:

$$U_{\hat{R}^n}(\hat{\mathbf{R}},\hat{\mathbf{P}})-\boldsymbol{\lambda}^T\boldsymbol{I}_{\hat{R}^n}(\hat{\mathbf{R}},\hat{\mathbf{P}})$$
$$-\boldsymbol{\mu}^T\boldsymbol{E}_{\hat{R}^n}(\hat{\mathbf{R}},\hat{\mathbf{P}})-\nu_n=0, \quad n\in\mathcal{N} \quad (11)$$

$$U_{\hat{P}^n}(\hat{\mathbf{R}},\hat{\mathbf{P}}) - \boldsymbol{\lambda}^T \boldsymbol{I}_{\hat{P}^n}(\hat{\mathbf{R}},\hat{\mathbf{P}}) - \boldsymbol{\mu}^T \boldsymbol{E}_{\hat{P}^n}(\hat{\mathbf{R}},\hat{\mathbf{P}}) + t_n = 0, \quad n \in \mathcal{N} \quad (12)$$

where $F_x(.)$ refers to the derivative of a function $F(.)$ w.r.t. the variable x, e.g., $U_{\hat{P}^n}(\hat{\mathbf{R}},\hat{\mathbf{P}}) = \partial U(\hat{\mathbf{R}},\hat{\mathbf{P}})/\partial \hat{P}^n$.

P3. Solve the per-tone problems for each tone k:

$$\max_{\{\mathbf{s}_k \in \mathcal{S}_k\}} \sum_{n\in\mathcal{N}} \nu_n b_k^n(\mathbf{s}_k) - \sum_{n\in\mathcal{N}} t_n s_k^n \quad (13)$$

This non-convex problem in dimension N can be solved using existing per-tone solutions, e.g., [1–3, 6, 8].

From the three decomposed subproblems above, we can readily develop the procedure for solving the unified green DSL formulation (9), as is described in the algorithm **General-Green-DSL**. The functions $\boldsymbol{f}$ and $\boldsymbol{h}$ can be standard subgradient updates, i.e., $\boldsymbol{\nu} = \boldsymbol{\nu} + \epsilon(\hat{\mathbf{R}} - \mathbf{R})$ and $\boldsymbol{t} = \boldsymbol{t} + \epsilon(\mathbf{P} - \hat{\mathbf{P}})$, where ϵ refers to the step size for the subgradient updates. We prove that the algorithm converges to a point which is either a locally or globally optimum, depending on the particular DSM algorithm used in **P3**. We skip its convergence proof due to space limitation.

General-Green-DSL

1: Initialize $\hat{\mathbf{R}}, \hat{\mathbf{P}}, \{\mathbf{s}_k \in \mathcal{S}_k, k \in \mathcal{K}\}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{t}, \boldsymbol{\nu}$
2: **repeat**
3: $\quad \boldsymbol{\lambda} = [\boldsymbol{\lambda} + \epsilon(\boldsymbol{I}(\hat{\mathbf{R}},\hat{\mathbf{P}}))]^+$
4: $\quad \boldsymbol{\mu} = \boldsymbol{\mu} + \epsilon(\boldsymbol{E}(\hat{\mathbf{R}},\hat{\mathbf{P}}))$
5: $\quad \boldsymbol{\nu} = \boldsymbol{f}(\hat{\mathbf{R}},\hat{\mathbf{P}}, \{\mathbf{s}_k \in \mathcal{S}_k, k \in \mathcal{K}\}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{t}, \boldsymbol{\nu})$
6: $\quad \boldsymbol{t} = \boldsymbol{h}(\hat{\mathbf{R}},\hat{\mathbf{P}}, \{\mathbf{s}_k \in \mathcal{S}_k, k \in \mathcal{K}\}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{t}, \boldsymbol{\nu})$
7: $\quad$ Obtain $\hat{\mathbf{R}}$ and $\hat{\mathbf{P}}$ by solving system (11) and (12)
8: $\quad \forall k \in \mathcal{K}$: Solve per-tone problem (13)
9: **until** convergence

5. SIMULATION RESULTS

We consider a practically highly relevant ADSL scenario as shown in Figure 1, where for illustrative purpose we present the 2-user dynamics. This is a so-called near-far scenario, which is known to be a challenging scenario, where DSM can make a substantial difference. More extensive scenario simulations will be presented in the journal version. The following realistic parameters settings are assumed in the channel simulator. The twisted pair lines have a diameter of 0.5 mm (24 AWG). The maximum modems' total transmit power $P^{n,\text{tot}}$ is 20.4 dBm. The SNR gap Γ is 12.9 dB. The tone spacing Δ_f is 4.3125 kHz. The DMT symbol rate f_s is 4 kHz. For solving the per-tone problem (13), we used the MS-DSB algorithm that is shown to converge to the globally optimal solution for this scenario [8].

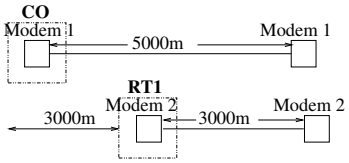


Figure 1: 2-user near-far ADSL scenario

Figure 2(a) shows the trade-off between the *normalized* sum data rate performance (w.r.t the maximum unweighted sum rate $\sum_n R^{n,o}$ for full power usage), and the *normalized* greening measured by the actual consumed power divided by the total maximum available power (i.e., $\sum_n P^n / \sum_n P^{n,\text{tot}}$), for the six different greening methods in Section 4.1. We generally observe logarithmic curves, implying that significant power savings can be achieved with only small degradations in data rate performance. Different greening methods pay a different price of greening (i.e., loss in sum rate data rate performance due to greening). Greening 3A is the best in terms of sum rate performance, i.e., 50% of system power is saved while still achieving 93% of data rate performance. However, Greening 3A is intuitively the most unfair, since it contains no mechanisms to steer towards fair transmit power and/or data rate allocations. Other greening methods have worse sum rate performance, where in particular that of Greening 1 (energy-fair) gives the smallest sum rate. Note that for Greenings 3B and 3C, only one simulation point is shown rather than a parametric curve, due to their dependency on the chosen weight δ in (7). Different values for this weight lead to different trade-offs, and it is not straightforward to tune this weight δ in order to arrive at a specific tradeoff point. This extra tuning for a good trade-off can be regarded as a pitfall of Greenings 3B and 3C from the perspective of green system design.

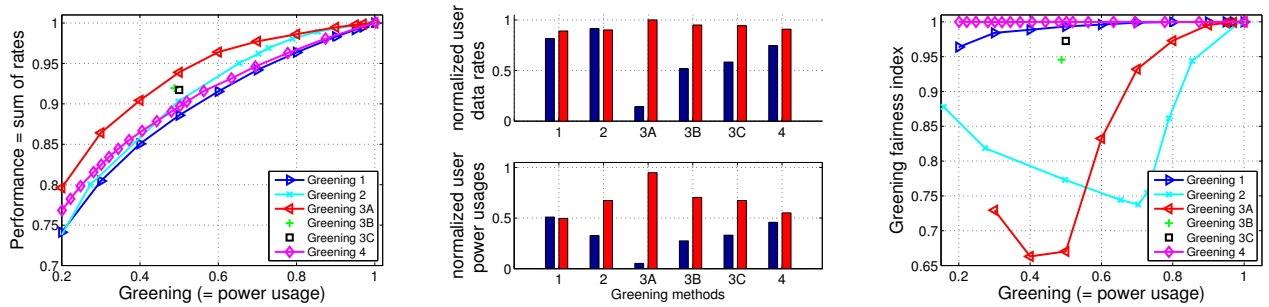
Figure 2(b) shows the distribution of the *normalized* data rates (w.r.t. the full power data rate $R^{n,o}$ for each user n) and *normalized* transmit powers (w.r.t. the full power $P^{n,\text{tot}}$ budget of each user n) across the two users for different greening methods when 50% greening is applied. We observe that Greening 3A allocates the data rates and the transmit powers very unfairly over the users, i.e., user 2 dominates over user 1 in terms of transmit power and data rate. By definition, Greening 1 equalizes the normalized energy as it proportionally allocates its transmit powers over the users. Greening 2 equalizes the normalized data rate, but has uneven allocation of transmit powers. Greenings 3B and 3C succeed in obtaining relatively better fairness in terms of transmit powers as well as data rates than Greening 3A. This is due to the addition of the fairness term into their objective functions. However, the price of this is a reduced sum data rate performance as seen in Figure 2(a), compared to Greening 3A. Finally, Greening 4 reduces its data rates proportionally to its transmit powers for all modems as it was designed for.

To further quantitatively evaluate the fairness of greening methods, a definition of fairness measure is needed. A good candidate is a measure that jointly considers energy and rate fairness in its definition. To that end, we propose the following definition of *greening fairness index* $\mathcal{F}$:

$$\mathcal{F} = \frac{1}{N-1}\Big(\frac{(\sum_{n\in\mathcal{N}} x_n)^2}{\sum_{n\in\mathcal{N}} x_n^2} - 1\Big), \quad (14)$$

with $x_n = (R^n/R^{n,o})/(P^n/P^{n,o})$ and $\{\mathbf{R}^o, \mathbf{P}^o\}$ denotes the point on the boundary of the rate region without greening. $\mathcal{F} = 1$ when all users have the same ratio between data rate decrease and power decrease, and approaches 0 as these ratios start to deviate from each other. Figure 2(c) shows the trade-off between the greening fairness index $\mathcal{F}$ and power reduction due to greening for the proposed six methods. The key messages are as follows:

- Greening 4 is 100% fair w.r.t. $\mathcal{F}$, since it was constructed by considering both energy and data rate in the first place.



(a) *Normalized* sum data rate performance vs. greening for different greening methods

(b) *Normalized* data rates (top) and *normalized* transmit powers (bottom) for two modems (blue: modem 1, red: modem 2) for different greening methods when 50% greening is applied

(c) Greening fairness index $\mathcal{F}$ (14) vs. greening for different greening methods

Figure 2: Simulation Results

- Energy-fair Greening 1 is also quite fair, whereas rate-fair Greening 2 is unfair.
- Sum rate greening 3A is very unfair. However, by adding the fairness terms to Greening 3A, i.e. Greening 3B and 3C, the fairness behavior, w.r.t. $\mathcal{F}$, becomes much better, i.e. from 67% to more than 94%.

In summary, Green DSL can generally lead to large energy savings with only minor data rate performance degradation. However one should carefully choose its greening strategy that significantly affects fairness in the data rates and the amount of energy savings. Some greening strategies can lead to very unfair allocations in terms of data rates and/or transmit powers, which can even put some users on weak DSL lines out-of-service. The application of fair greening technology can prevent that, but the price to pay is possibly a further loss in sum rate performance.

6. CONCLUSION

In order for green DSL to be deployed, Internet Service Providers must ensure certain notions of fairness across the interfering users, when allocating the tradeoff between high data rate and low energy consumption. There is no single universally agreed notion of fairness in greening, and this paper explores four alternative approaches. Their unified representation can be solved effectively by the General-Green-DSL algorithm. They present a range of choices in the three way tradeoff among sum data rate, total energy consumption, and the fairness index. In particular, Method 4 guarantees a proportional relationship between performance and greening across the users, while maintaining a reasonable data rate.

7. REFERENCES

[1] R. Cendrillon, J. Huang, M. Chiang, and M. Moonen. Autonomous spectrum balancing for digital subscriber lines. *IEEE Trans. on Signal Processing*, 55(8):4241–4257, 2007.

[2] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, and T. Bostoen. Optimal multiuser spectrum balancing for digital subscriber lines. *IEEE Trans. Communications*, 54(5):922–933, 2006.

[3] R. Lui and W. Yu. Low-complexity near-optimal spectrum balancing for digital subscriber lines. In *Proceedings of IEEE ICC*, 2005.

[4] Z. Q. Luo and S. Zhang. Dynamic spectrum management: Complexity and duality. *IEEE Journal of Selected Topics in Signal Processing*, 2(1):57–73, 2008.

[5] E. C. D. J. Paolo Bertoldi. European code of conduct for broadband equipment. Internet: `http://www.itu.int/dms_pub/itu-t/oth/09/05/T09050000010004PDFE.pdf`, 2008.

[6] J. Papandriopoulos and J. S. Evans. Low-complexity distributed algorithms for spectrum balancing in multi-user DSL networks. In *Proceedings of IEEE ICC*, 2006.

[7] T. Starr, J. M. Cioffi, and P. J. Silverman. *Understanding digital subscriber lines*. Prentice Hall, 1999.

[8] P. Tsiaflakis, M. Diehl, and M. Moonen. Distributed spectrum management algorithms for multi-user DSL networks. *IEEE Trans. on Signal Processing*, 56(10):4825–4843, 2008.

[9] P. Tsiaflakis, Y. Yi, M. Chiang, and M. Moonen. Green DSL: Energy-efficient DSM. In *Proceedings of IEEE ICC*, 2009.

[10] W. Yu and R. Lui. Dual methods for nonconvex spectrum optimization of multicarrier systems. *IEEE Trans. on Communications*, 54(7), July 2006.

[11] M. Wolkerstorfer, D. Statovci, and T. Nordstrm. Dynamic spectrum management for energy-efficient transmission in DSL. In *Proceedings of ICCS (International Conference on Communications Systems)*, 2008.

[12] W. Yu, G. Ginis, and J. Cioffi. Distributed multiuser power control for digital subscriber lines. *IEEE J. Sel. Area. Comm.*, 20(5):1105–1115, 2002.