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BRUTE: Energy-efficient User Association in Cellular Networks from Population Game Perspective

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***Abstract*—In this paper, we address the problem of associating mobile stations (MSs) with base stations (BSs) in an energy-efficient manner. We take a population game approach, which allows tractable analysis of many selfish mobiles without growing mathematical complexity. From our game-theoretical analysis, we prove that a simple power-dependent pricing by operators leads a Nash equilibrium to be equal to the optimal solution of a social optimization problem (i.e., no price-of-anarchy). We study three evolution dynamics of associating MSs, each expressed as a differential equation, all of which provably and/or numerically converge to the Nash equilibrium. Based on several considerations regarding implementation of association algorithms in practice, we found that asynchronicity and fast load tracking are the key components to practical algorithms. Motivated by this, we propose a practical energy-efficient user association mechanism, named BRUTE. To evaluate the performance of BRUTE, we implement a cellular network simulator using an event-driven simulator, SimPy, and perform extensive simulations under various scenarios including a real BS topology in UK. Our simulation results show that BRUTE outperforms other conventional user association techniques.**

***Index Terms*—User association, population game, evolutionary dynamics, load balancing, cellular networks;**

I. Introduction

IN response to high data demand in cellular systems, *user association*, the problem of associating a mobile station (MS) [1] with an appropriate base station (BS) is of prime importance. It has been evidenced in literature that a simple approach of connecting an MS to the BS providing the highest received signal strength has substantial performance degradation due to its load-agnostic behavior. In fact, the user population in a cell has significant impact on the actual individual MS throughput, thereby many load-aware association schemes have been proposed so far [1]–[12].

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[1] We use 'user', 'MS', and 'player' interchangeably throughout this paper.

In addition to performance, energy-efficiency in wireless networks is another important metric. Specifically, recent interests in greening such as the CO_2's potential harms (e.g., global warming) to the environment as well as the economic issues recently motivate a surge of energy-efficient research. There are many components to save energy in cellular networks, ranging from cooler and power amplifiers to dynamic switching on/off of BSs. User association also highly affects energy consumption in the network, energy and load aware user association is getting more important. There exists, however, a complex interplay between energy-efficiency and performance (such as throughput or delay), often showing a tradeoff. This is because high performance requires load balancing of MSs, whereas energy efficiency increases when MSs are associated with nearby BSs, which often results in load-imbalance.

In this paper, we study an energy-efficient user association problem from a population game-theoretic perspective. Population game [13] groups the entire MSs into a *finite* number of classes of infinitesimal MSs having similar attributes, e.g., the set of connectable BSs, their link conditions, and the spatial traffic distribution. This enables us to have a mathematically tractable framework without growing mathematical complexity and easily obtain the implications into distributed BS association mechanisms. Our model uses a flow-level dynamic where data traffic is initiated at random and its workload is also random, so that after a random amount of sojourn time in the system it leaves. This flow-level dynamic seems to provide more practical intuitions and results than the statically backlogged setting often taken in other researches. In our flow-level dynamic, we model spatially heterogeneous traffic distribution and also capture signal degradation incurred by interference from other BSs.

In our game, we model the payoff function by the combination of the selfish performance objective of users and the cost for using BSs' energy, where a user's selfish performance objective is described by the delay performance conditioned on the user's offering load. We first prove that the population game designed by the aforementioned payoff function becomes a potential game. In potential game, Nash equilibrium (NE) is characterized by the Karush-Kuhn-Tucker (KKT) condition of the potential function, offering an easy path to the equilibrium analysis. Then, we prove that the NE coincides with the socially optimal point, implying there is no price-of-anarchy. This remarkable result stems from a smart association pricing scheme, instilled as a cost part of user's payoff function.

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Next, we consider three kinds of evolutionary dynamics, (i) best response dynamic, (ii) replicator dynamic, and (iii) Brown-von Neumann-Nash (BNN) dynamic, each of which captures how mobiles evolve over time as the system state changes. As studied in literature, the best response and BNN dynamics provably converge to the NE. Unfortunately, the replicator dynamic may not converge to a stationary point that is NE, but under some reasonable conditions of initial points, we experimentally show that the replicator dynamic is also highly likely to converge to NE.

Finally, we aim at developing a practical energy-efficient algorithm inspired by the theoretical analysis mentioned above. Note that three dynamics were originally developed to model selfish players in population games, but we connect them to energy-efficient, distributed association control algorithms. To that end, we carefully examine the convergence speed and the required complexity (e.g., message passing) of three dynamics, when converted to practical association protocols, from which we propose an association algorithm motivated by the best-response dynamic due to its fast convergence and small complexity. In fact, we show that a distributed association algorithm [9], [10] developed from an optimization's perspective can be "almost" reverse-engineered by our game-theoretic approach, with more practical advantages.

Our new approach strengthens the practicality of the user association because the proposed user association achieves the socially optimum without time-scale separation assumption, which becomes possible by a traffic estimation technique. Note that the prior work based on flow-level dynamics such as [9], [10] assumed time-scale separation between BS load estimation and actual BS association, which renders the problem more manageable, but differs from the practice. By performing discrete event simulations we verify that the proposed association algorithm, named **BRUTE** (Best Responding User association with Traffic Estimation), is highly efficient, in relation to the conventional user association scheme which only considers the received data rate when choosing BSs. Also, our proposed **BRUTE** outperforms the algorithm introduced in [10]—namely SYNC throughout the paper, which has exactly same payoff function as **BRUTE** but without traffic estimation.

Related work

Recently, the authors in [10] formulate an optimization problem that trade offs performance and energy efficiency, and study both energy-efficient association and dynamics BS on/off switching. They use a time-scale separation between association and on/off operations, enabling two different problems—the authors assumed that BS on/off operations run in much slower time scale than association, which enables two problems to be decoupled. The social objective function in our paper is equivalent to that in [10] without BS dynamic on/off switching. However, our paper significantly differs from [10] in that we approach the problem from the game-theoretic perspective. Specifically, using a population game framework, we consider a *finite* number of classes in describing different heterogeneous traffic characteristics and a discrete set of MS data rates; in [9], [10], it was assumed that there exist an infinite number of classes, and available data rates are continuous just for simplicity. However, this simplification does not capture the real systems well, and thus the resulting deterministic user association [9], [10] does not generally achieve optimality with a finite number of classes in practice. This is because the cell boundary should be a *region*, not a *line* as in [9], [10] when adaptive modulation and coding (AMC) is employed. We tackle user association problem with the finite class setting using the population game gheory, and investigate the optimality condition of the distributed algorithm in [9], [10] in Section VI.

There exists work, see e.g., [11], [14]–[22] that studies a BS/WLAN association problem in a game-theoretic setting. The authors in [14] used the user performance of UDP/TCP throughput with varying frame lengths over WLAN APS (access points), whereas we use the flow-level delay as a performance metric which depends on BS load. The authors in [11] suggested the general concave utility function, formulated a game, and proved that the total utility is maximized at the Nash equilibrium. The work in [15] studied a Stackelberg game between BS placement and user association. The main difference from the above lies in that we consider both energy-efficiency and flow-level dynamic using a population game. The authors in [16] considered airtime cost as the metric of the load of AP and formulated a non-cooperative game between WLAN stations, which has been extended to [17] that jointly solves AP association and channel assignment. The work [20], [22] took account of QoS of mobile users and modeled a matching game between cellular users and small cell base stations. The authors in [21] provided an analogy of uplink user association as college admission game between mobile users and BSs who have conflicting interests, where they solved the problem using matching theory and coalitional game. In [19], a two-tier game framework of cell selection and channel selection is proposed and they proved the existence of NE and convergence of a proposed dynamic algorithm. In [18], the evolutionary game approach was taken for user association problem in femtocell networks, where they assumed a logarithmic revenue and a linear payoff for the performance metric and used reinforcement learning technique in order to decentralize the evolutionary dynamics.

Other related work includes [23], which studies the load balancing problem among server farms (where a server can be considered as a BS in our case) using game theory. In [23], the authors assumed a fixed processing capacity of each server and the capacity does not depend on users. We model spatially heterogeneous users, and thus BS-user capacity should differ across users, which makes the problem much more challenging. There exists an array of research on BS load-balancing. Some of earlier studies assumed a centralized processor that establishes cell load-balancing [1]–[7]. Due to its weakness in terms of scalability and flexibility, the distributed algorithms were proposed [8]–[10]. We refer the readers to [24] for a list of networking problems analyzed by population game theory. Greening with focusing on dynamic BS on/off switching has been studied in [10], [25]–[28].

There also exist the work based on the population game theory which studied energy-efficiency issues in wireless net-

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works [29]–[32]. Mériaux et al. [31], [32] investigated a power control problem in CDMA-type wireless networks using the mean field game theory. They took account of energy constraints in the payoff function, and solved the problem using mean field approximation under the assumption of the large-scale system. Tembine, Altman et al. [29], [30] formulated the stochastic population games to analyze a non-cooperative power control scheme in ALOHA-type wireless networks. The authors considered energy constraints by separating different battery levels of wireless nodes, and proved the existence of equilibrium point. All of these works mainly investigated non-cooperative situations when the end users behave to maximize its own throughput and save the battery power, in ALOHA/CDMA-type wireless systems. Our work differs from those existing works in the fact that we focused on the energy savings of the infrastructures nodes, not that of the end users. With a proper load balancing algorithm inspired from the population game theory, which trade offs the flow-level performance and the energy efficiency, we develop a practical energy-efficient user association algorithm.

II. Preliminaries: Population Game

Basic concepts. We briefly provide the basics of population game, which we refer the readers to [13] for more details. A *population game* F is defined by the *society* of continuous mass of user groups called *classes*. Denote the set of classes by $\mathcal{Q}$ and the number of classes by Q. Each class $q \in \mathcal{Q}$ has continuous *mass* d^q. Each class q has its own strategy sets $\mathcal{S}^q = \{1, ..., S^q\}$. A single entity in the class is called *player*, and each player in class q selects its own strategy among the strategy set $\mathcal{S}^q$. The *state* of class q is defined as its distribution of strategic decisions, denoted as $y^q = [y_1^q, ..., y_{S^q}^q]$, where y_i^q represents the mass of players in class q who plays strategy $i \in \mathcal{S}^q$. The set of states of class q is denoted as $\mathcal{Y}^q = \{y^q \in \mathbb{R}_+^{S^q} | \sum_{i\in\mathcal{S}^q} y_i^q = d^q\}$. The social state $\boldsymbol{y} = [y^1, ..., y^Q]$ is simply a Cartesian product of the class states. Again, the set of all possible social states is denoted as $\mathcal{Y} = \prod_{q\in\mathcal{Q}} \mathcal{Y}^q$. The marginal payoff function F_i^q per unit mass of each class q for each strategy i is defined on each social state. Thus, we have a collection of marginal payoff functions $F = (F_i^q : i \in \mathcal{S}^q, q \in \mathcal{Q})$. We also use F to name a population game for notional simplicity. Each player in each class receives its payoff depending on its own strategic decision. The aggregate payoff of class q is $\sum_{i\in\mathcal{S}^q} y_i^q F_i^q(\boldsymbol{y})$.

Best response and Nash equilibria. A solution concept generally used in game theory is *Nash equilibrium*. This notion is also used in population games. We first define the concept of *best response correspondence*, which means a set of selfishly optimal strategies given a social state. In class q, the pure best response correspondence $b^q : \mathcal{Y} \to \mathcal{S}^q$ is defined by $b^q(\boldsymbol{y}) = \arg\max_{i\in\mathcal{S}^q} F_i^q(\boldsymbol{y})$. Also, the mixed best response correspondence for class q is defined as $B^q(\boldsymbol{y}) = \{x^q \in \mathcal{Y}^q : x_i^q > 0 \to i \in b^q(\boldsymbol{y})\}$.

Definition 1: A social state $\boldsymbol{y} \in \mathcal{Y}$ is a *Nash equilibrium* of the population game F if every player in the society is choosing the best response of $\boldsymbol{y}$, i.e., the set of all Nash equilibria $\mathrm{NE}(F)$ is:

$$\mathrm{NE}(F) = \{\boldsymbol{y} \in \mathcal{Y} : y^q \in B^q(\boldsymbol{y}) \text{ for all } q \in \mathcal{Q}\}.$$

Potential game. If the payoff function F has a special form, the characterization and analysis of Nash equilibria becomes much more tractable.

Definition 2: A population game F is a *potential game* if there exists a function in C^1 space[2] called *potential function* $\Phi : \mathcal{Y} \to \mathbb{R}$, satisfying $\frac{\partial \Phi}{\partial y_i^q}(\boldsymbol{y}) = F_i^q(\boldsymbol{y})$, i.e., $\nabla\Phi(\boldsymbol{y}) = F(\boldsymbol{y})$, for all $\mathbf{y} \in \mathcal{Y}, i \in \mathcal{S}^q$, and $q \in \mathcal{Q}$.

The concept of potential game has first appeared in [33]. This potential game has been extended in the large population context by [34], which we apply to our problem in this paper. Note that the potential game in this paper is an exact potential game, since the derivative of potential function is exactly equal to the payoff function. There are other kinds of potential games, such as ordinal potential game, which is the game where only the signs of the derivative of the potential function and the payoff function match. In the potential game, the strategic improvement of users increases potential function. Thus, at the local maxima of the potential function, there exist no incentives for each player to deviate from its own decision. In other words, the local maximum of the potential function is equivalent to a Nash equilibrium, as summarized as the following theorem [13]:

Theorem 1 ([13]): If a population game F is a potential game with the potential function Φ, then $\mathrm{NE}(F) = \mathrm{KKT}(\Phi)$, where $\mathrm{KKT}(\Phi)$ is the set of points satisfying the KKT condition of Φ.

III. System Model

Network and Users. We consider a cellular network consisting of a set $\mathcal{S}$ of BSs. The society corresponds to the set $\mathcal{Q}$ of all users, composed of a finite set of classes, where a class q is a population of users who commonly share (i) the set $\mathcal{S}_q$ of BSs allowing association to the entities of class q and (ii) the link capacity from each of such BSs, (iii) the traffic characteristic. As mentioned earlier, we assume that each class q has a

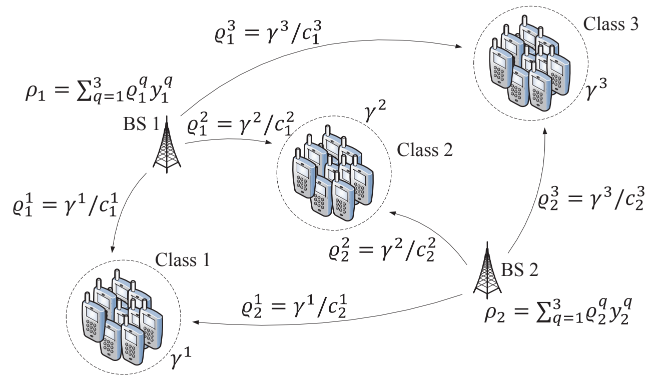

Fig. 1. System model and notations.

[2]This is the space of continuously differentiable functions.

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continuous mass of users, and the class-q mass is denoted by d^q. Let $\mathcal{Q}_s$ be the set of classes that can be potentially associated with the BS $s \in \mathcal{S}$. Here y_i^q denotes the mass of users in a class q who select BS i. Note that $\sum_i y_i^q = d^q$ and the overall selection profile $\boldsymbol{y}$ determines the overall system state.

Traffic, Capacity, and Load. All users in class q have a Poisson arrival of file transfer requests with rate λ^q, and each file size is independently distributed with mean $1/\mu^q$. Thus, the total request rate of class q is $\lambda^q d^q$. Let the traffic load density of class q per unit mass be $\gamma^q = \lambda^q/\mu^q$. As mentioned earlier, we assume that all users in the same class receive the same link capacity from each BSs. Denote c_i^q the link capacity that each user in class q can achieve from the BS i. Note that c_i^q may differ among all pairs of user classes and BSs and c_i^q can capture the inter-cell interference as well. The *system-load density* ϱ_i^q is defined as the time fraction required by BS i to serve the request from the unit mass of the class q, i.e., $\varrho_i^q = \gamma^q/c_i^q$, where it is assumed $\min_i \varrho_i^q < \infty$, which means there exists at least one BS which provides positive link capacity to the class q and thus can serve the request from the class q. Let ρ_i be the *load* of the BS i. The load ρ_i is represented as the sum of the traffic loads in BS i from all classes, i.e., $\rho_i = \sum_{q\in\mathcal{Q}} \varrho_i^q y_i^q$. The ρ_i can be interpreted as the fraction of time needed in BS i to serve the entire incoming traffic. The load ρ_i should be less than 1 in order to make the system stable, which is assumed in this paper. Fig. 1 visually explains our system model.

BS Energy Model. We model BSs' energy consumption by a combination of the static power consumed whenever a BS is turned on and the load-dependent power, where each portion is tunable by a parameter, as stated next:

$$\text{Energy consumption of BS } i = (1-q_i)\rho_i P_i + q_i P_i, \qquad (1)$$

where P_i is the amount of energy of BS i when fully utilized and $0 \le q_i \le 1$ is the parameter quantifying the portion of the static power at BS i. For example, the case $q_i = 0$ corresponds to the BS that is entirely *energy-proportional*. This BS energy model is chosen to render our analysis generically applicable. Note that in the current practice, a typical UMTS BS consumes 800-1500W for static power and 20-40W for RF output power [10], and q_i is not close to 0; the range of q_i is roughly 0.5–0.8 in 3G cellular networks [35].

IV. Association Game and Equilibrium Analysis

We now define a population game, called *association game*, by completing the model of (marginal) payoff function for each class. Prior to the game description, we first present a social optimization problem that may be intended to be solved by a regulator, e.g., an MNO (Mobile Network Operator). We later compare the equilibrium of the defined game and the optimal solution of the social optimization problem.

A. Social Objective

Consider the following optimization problem:

$$\begin{aligned} &\text{maximize} && \Phi(\boldsymbol{y}) = \Phi_{F,\alpha}(\boldsymbol{y}) + \eta\Phi_G(\boldsymbol{y}) \qquad (2)\\ &\text{subject to} && \rho_i = \sum_{q\in\mathcal{Q}} y_i^q \varrho_i^q < 1 \text{ for all } i \in \mathcal{S}\\ &&& \text{and} \sum_{i\in\mathcal{S}^q} y_i^q = d^q \text{ for all } q \in \mathcal{Q}, \end{aligned}$$

where the term $\Phi_{F,\alpha}(\boldsymbol{y})$ corresponds to *flow-level performance*, the term $\Phi_G(\boldsymbol{y})$ represents the amount of *energy consumption*, and $\eta \ge 0$ is the parameter that trades off those two metrics.

In the above, the flow level performance characterizes the average performance that a typical flow experiences, where a flow may correspond to a single TCP session or a single file transfer. To be more precise about the flow-level performance, the performance term $\Phi_{F,\alpha}(\boldsymbol{y})$ in (2), we take the approach in [9] that parameterizes the flow-level efficiency with α:

$$\Phi_{F,\alpha}(\boldsymbol{y}) = \begin{cases} -\sum_{i\in\mathcal{S}} \dfrac{(1-\rho_i(\boldsymbol{y}))^{1-\alpha}-1}{\alpha-1}, & \alpha \neq 1\\ -\sum_{i\in\mathcal{S}} \log\left(\dfrac{1}{1-\rho_i(\boldsymbol{y})}\right), & \alpha = 1. \end{cases} \qquad (3)$$

The parameter α is called *degree of load balancing*. For $\alpha = 0$, the function becomes $\sum_{i\in\mathcal{S}} \rho_i$, hence the users have *rate-optimal* behavior. For $\alpha = 2$, corresponding to *delay-optimal*, the function becomes $\sum_{i\in\mathcal{S}} \frac{\rho_i}{1-\rho_i}$, which is proportional to the average delay of M/GI/1 multi-class processor sharing queue [36]. The second term in (2) represents the total energy consumption of BSs (simply corresponding to a cost term), given by the summation of consumed energy over all BSs:

$$\Phi_G(\boldsymbol{y}) = -\sum_{i\in\mathcal{S}} \left[(1-q_i)\rho_i(\boldsymbol{y})P_i + q_i P_i\right]. \qquad (4)$$

Note that we put negative signs to both terms in (2) simply to formulate the target optimization a *maximization* problem.

B. User Association Game Formulation

We now design the association game for which we define the marginal payoff function for each class q and the strategies available to the class q. Note that the marginal payoff function is interpreted as the payoff obtained by the newcomers in the corresponding class when all other users' strategies are given in population game theory. We consider the following form of the marginal payoff function:

$$\begin{aligned} F_i^q(\boldsymbol{y}) &= -\left[\frac{\varrho_i^q}{(1-\rho_i(\boldsymbol{y}))^\alpha} + \eta P_i \varrho_i^q (1-q_i)\right]\\ &= -\varrho_i^q\left[(1-\rho_i(\boldsymbol{y}))^{-\alpha} + \eta P_i(1-q_i)\right]. \qquad (5) \end{aligned}$$

The above payoff function consists of two major terms: (i) selfish flow-level utility and (ii) power pricing.

(i) *Selfish flow-level utility.* The first term of (5) denotes the selfish utility motivated by the selfish flow-level performance. For $\alpha = 0$, the first term becomes $\varrho_i^q (= \gamma^q/c_i^q)$, directing users to selfishly prefer the BSs providing high rate without considering the offered load in the associating BS. For $\alpha = 1$, this term becomes proportional to the *conditional* delay experienced by the users in the class q, where the conditional delay means the delay experienced

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by an user conditioned on associating with a particular BS, i in this case (see [9], [23] that use a similar notion of conditional delay under different models for different purposes). As α grows, users increasingly take into consideration the BS loads in association, as the payoff function decades more sharply with increasing ρ_i.

(ii) *Power pricing.* The second term corresponds to the consumed energy of BS i to serve the users in class q. Note that this term does not depend on the social state, implying that the cost of associating with a particular BS is independent of other class' offered load. Recall that $1-q_i$ is the portion of load-dependent, consumed energy. Thus, $P_i \varrho_i^q (1-q_i)$ corresponds to the consumed energy only by class q, that can be interpreted as the *price* that an user in class q should pay to use BS i's power resource. An interesting feature is that when $q_i = 1$ (energy unproportional), there is no incurred power cost in this marginal payoff function.

C. Equilibrium Analysis and Price-of-Anarchy

In this subsection, we provide the equilibrium analysis of our game. Three main features of our interests are: existence, uniqueness, and Price-of-Anarchy (PoA) [37] of the equilibrium (i.e., NE). Let $y^\star$ and y^{NE} be the socially optimal solution of (2) and an NE (if it exists), respectively. Following [37], the PoA is defined as the ratio between the worst Nash equilibrium point and the social optima. Since $\Phi(\boldsymbol{y})$ is strictly concave respect to ρ, all NEs in our model give the same value of $\Phi(\boldsymbol{y})$. Thus we henceforth denote the PoA simply as $\Phi(\boldsymbol{y}^\star)/\Phi(\boldsymbol{y}^{\text{NE}})$. In many cases, it is quite challenging and mathematically complex to analyze those three features, especially when the game has a large degree of couplings. However, our game is provably a potential game, opening an easy path to the analysis, as we will henceforth discuss in this subsection.

We first prove that our association game is a potential game. Lemma 1 states that the social objective function in (2) is the potential function.

Lemma 1: The objective function $\Phi(\boldsymbol{y})$ in (2) is a potential function of the population game with the marginal payoff function (5).

Proof: From Definition 2, it suffices to check $\frac{\partial \Phi}{\partial y_i^q}(\boldsymbol{y}) = F_i^q(\boldsymbol{y})$. For the case of $\alpha = 1$,

$$\begin{aligned} &\frac{\partial \Phi}{\partial y_i^q}(\boldsymbol{y}) \\ &= -\frac{\partial}{\partial y_i^q}\left[\sum_{i\in\mathcal{S}} \log\left(\frac{1}{1-\rho_i}\right) + \eta \Phi_G(\boldsymbol{y})\right] \\ &= -\left[(1-\rho_i)\cdot\frac{1}{(1-\rho_i)^2}\cdot\frac{\partial \rho_i}{\partial y_i^q} + \eta(1-q_i)P_i\frac{\partial \rho_i}{\partial y_i^q}\right] \\ &= -\varrho_i^q\left[(1-\rho_i)^{-1} + \eta P_i(1-q_i)\right] = F_i^q(\boldsymbol{y}). \end{aligned}$$

Similarly, for the case of $\alpha \neq 1$,

$$\begin{aligned} &\frac{\partial \Phi}{\partial y_i^q}(\boldsymbol{y}) \\ &= -\frac{\partial}{\partial y_i^q}\left[\sum_{i\in\mathcal{S}} \frac{(1-\rho_i)^{1-\alpha}-1}{\alpha-1} + \eta \Phi_G(\boldsymbol{y})\right] \\ &= -\left[\frac{1-\alpha}{\alpha-1}\cdot(1-\rho_i)^{-\alpha}\cdot\left(-\frac{\partial \rho_i}{\partial y_i^q}\right) + \eta(1-q_i)P_i\frac{\partial \rho_i}{\partial y_i^q}\right] \\ &= -\varrho_i^q\left[(1-\rho_i)^{-\alpha} + \eta P_i(1-q_i)\right] = F_i^q(\boldsymbol{y}). \end{aligned}$$

■

Lemma 2: The potential function $\Phi(\boldsymbol{y})$ is concave in $\boldsymbol{y}$.

Proof: It has been proved by [10] that Φ is concave in $\boldsymbol{\rho}$. Since Φ is non-increasing in $\boldsymbol{\rho}$, and $\boldsymbol{\rho}$ is concave over $\boldsymbol{y}$ (since $\boldsymbol{\rho}$ is linear combination of the components of $\boldsymbol{y}$). From concavity-preserving operations, the composition of two functions $\Phi(\boldsymbol{\rho})$ and $\boldsymbol{\rho}(\boldsymbol{y})$ becomes $\Phi(\boldsymbol{y})$ which is concave in $\boldsymbol{y}$. ■

From Theorem 1, the NE of our game can be easily characterized by KKT condition of the potential function Φ. Therefore, all NE points satisfy the KKT condition of the potential function Φ. Lemma 2 guarantees that the local maxima (i.e., NEs) are also the global maxima of the potential function Φ. Note that the uniqueness of NE is not guaranteed, which means there can be multiple association scenarios at NEs.

Theorem 2: Our association game defined by the marginal payoff function (5) has PoA value one, i.e., no price-of-anarchy.

Proof: Lemma 2 implies that the optimization problem (2) is indeed convex optimization problem, and has zero duality gap. Thus, the points satisfying KKT condition of the problem globally maximize the social objective function (2). Also, from Lemma 1, the social objective function is a potential function. Therefore, from Theorem 1, the NE of our game coincides with the point satisfying KKT condition of the potential function. Hence, it is guaranteed that NE actually exists (derived from KKT condition), and all NE points globally maximizes the social objective function. ■

V. Evolutionary Dynamics

In this section, we consider *evolutionary dynamics* to study how users' association evolves over time and converges (if it does). We consider three popular dynamics in the area of population games, discuss their convergence to a NE. An evolutionary dynamic is expressed by a differential equation $\dot{\boldsymbol{y}} = V(\boldsymbol{y})$, where $V : \mathcal{Y} \to \mathbb{R}$ is a state-dependent vector field which defines the drift of the social state. Here we introduce three kinds of well-known evolutionary dynamics [13], the replicator dynamic, Brown-von Neumann-Nash (BNN) dynamic, and the best response (BR) dynamic. We will discuss the convergence behavior of three dynamics as well. Then we numerically compute differential equations of three dynamics in a simple two-cell scenario. Although two dynamics are not ready to be implemented as practical association algorithms, it is worthwhile to investigate the convergence and convergence speed of each dynamic, because they motivate practical association algorithm, as presented in Section VI.

A. Three Evolutionary Dynamics

Replicator dynamic. The first dynamic widely used in evolutionary dynamics is replicator dynamic. Its basic idea is to form a drift vector based on the average payoff of the

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corresponding class, where the drift is made, so that each user prefers a strategy with larger *excess payoff* (i.e., the difference between the current strategy's payoff and the average payoff). The replicator dynamic is described as:

$$\dot{y_i^q} = V(\boldsymbol{y}) = y_i^q \left(F_i^q(\boldsymbol{y}) - \frac{1}{d^q} \sum_{i \in \mathcal{S}^q} y_i^q F_i^q(\boldsymbol{y}) \right), \quad (6)$$

where the term $F_i^q(\boldsymbol{y}) - \frac{1}{d^q} \sum_{i \in \mathcal{S}^q} y_i^q F_i^q(\boldsymbol{y})$ corresponds to the excess payoff of the strategy i in class q. The replicator dynamic is an instance of *imitative protocols*. In other words, at each update epoch, each user in the class randomly encounters another user, called opponent. If the payoff of the opponent exceeds the user's own payoff, then the user selects the opponent's strategy with probability proportional to the payoff difference among two encounters. Replicator dynamic captures the strategy popularity as well as the excess payoff of each strategy in the sense that the strategy drift is proportional to both the excess payoff of the strategy and the number of users playing the strategy.

Brown-von Neumann-Nash (BNN) dynamic. The second dynamic is Brown-von Neumann-Nash (BNN) dynamic. For ease of exposition, we first define a variable k_i^q to be the maximum of the excess payoff and zero: $k_i^q \triangleq \max \left\{ F_i^q(\boldsymbol{y}) - \frac{1}{d^q} \sum_{i \in \mathcal{S}^q} y_i^q F_i^q(\boldsymbol{y}), 0 \right\}$. Then, BNN dynamic is expressed as:

$$\dot{y_i^q} = V(\boldsymbol{y}) = d^q k_i^q - y_i^q \sum_{i \in \mathcal{S}^q} k_i^q. \quad (7)$$

The intuition behind BNN dynamic is that at each update epoch, each user randomly picks a strategy and compares its payoff with the average payoff. If the payoff of the chosen strategy exceeds the average payoff, the user changes its own strategy with probability proportional to the excess payoff.

Best response dynamic. Finally, we investigate the best response (BR) dynamic. In the BR dynamic, at a given social state each user attempts to select its strategy that gives a maximum payoff. i.e.,

$$i^q \in b^q(\boldsymbol{y}) = \arg\max_{j \in \mathcal{S}^q} F_j^q(\boldsymbol{y}) \quad (8)$$

where i^q stands for strategy selection which gives best response for a user in class q, and $b^q(\boldsymbol{y})$ is the pure best response correspondence set given social state $\boldsymbol{y}$. When every infinitesimal user in class q behaves like (8), the overall strategy vector $\boldsymbol{y}^q$ tends to drift to the mixed best response correspondence $B^q(\boldsymbol{y})$ of the social state $\boldsymbol{y}$ (see Section II). Mathematically, BR dynamic is expressed by the map from each state to the differential inclusion:

$$\dot{y^q} \in V^q(\boldsymbol{y}) = B^q(\boldsymbol{y}) - y^q. \quad (9)$$

B. Convergence and Comparison

This section is devoted to summarizing the convergence of three dynamics and discussing their differences in convergence speed in the context of the association game. Convergence of three dynamics has been well studied in literature. We refer the readers to [13], [38], [39] for more details. We first define a notion of *positive correlation (PC)* related to a sufficient condition under which an evolutionary dynamic converges to NE.

Definition 3: $\dot{\boldsymbol{y}} = V(\boldsymbol{y})$ is positively correlated if

$$\begin{aligned} & V(\boldsymbol{y}) \cdot F(\boldsymbol{y}) \\ & = \sum_{q \in \mathcal{Q}} \sum_{i \in \mathcal{S}^q} F_i^q(\boldsymbol{y}) V_i^q(\boldsymbol{y}) > 0 \text{ whenever } V(\boldsymbol{y}) \neq 0. \end{aligned}$$

Positive correlation states that the drift rate and the payoff values are positively correlated. In potential games, if the dynamic satisfies PC then the potential function becomes *Lyapunov function*; the potential function Φ acts as a (global) Lyapunov function of the dynamic, since for all solution trajectories $\boldsymbol{y}_t$, (i) $\frac{d}{dt}\Phi(\boldsymbol{y}_t) = \nabla\Phi(\boldsymbol{y}) \cdot \dot{\boldsymbol{y}}_t = F(\boldsymbol{y}_t) \cdot V(\boldsymbol{y}_t) \geq 0$ and (ii) $V(\boldsymbol{y}_t) = 0$ whenever $\frac{d}{dt}\Phi(\boldsymbol{y}_t) = 0$ from PC. This means that all solution trajectories of the dynamic satisfying PC are non-decreasing until a stationary point, i.e., a point y with $\dot{\boldsymbol{y}} = V(\boldsymbol{y}) = 0$. Thus, all solution trajectories eventually converge to a stationary point. All three dynamics are known to be provably positive correlated.

However, all stationary points are not necessarily NEs, where a dynamic converges to either (i) a local maximum of the Lyapunov function or (ii) a boundary point of the set $\mathcal{Y}$. Another condition that enables a stationary point to be a NE is so-called *non-complacency* (NC) or *Nash stationarity*. The PC condition implies that all NE points are the stationary points and the NC condition guarantees that the NE points are *equivalent* to stationary points. Note that when the NC condition is not met, there exist stationary points that are not NE. The BNN and BR dynamics satisfy NC, allowing those two dynamics to converge to a NE. However, the replicator dynamic does not satisfy NC, which opens possibility of convergence to a stationary point that is not a NE, elaborated more in what follows: Note that in the replicator dynamic agents observe the strategies of other agents, and switch to the optimal strategy selected by other agents. Thus, a particular agent can only choose a strategy that is being played by other agent even when there exists a strategy which gives her a larger payoff. For example, when all agents start with the same strategy, the strategy profile will never change under the replicator dynamics. We refer the readers to [38] for the convergence of various evolutionary dynamics. Nonetheless, as shown in Section VII-A, the replicator dynamic converges to NE at least in the scenarios of our simulations, because when there exists a positive portion of players associating with each BS in the initial condition, which usually holds when each mobile selects its initial BS (uniformly) at random.

We now discuss the convergence speed of three dynamics. The BR dynamic does not perform any probabilistic operations and just switches to the strategy providing the largest payoff, whereas the replicator and BNN dynamics switch to better strategies probabilistically. This makes the convergence speed of BR incomparably faster than that of the other two. However, there is a chance of instability in the case of BR dynamic, which can be successfully resolved at the practical implementation stage. Also, we know that the convergence speed of replicator and BNN dynamics depends on initial

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conditions, as will be numerically verified in Section VII-A. If the initial distribution of the population is biased to one strategy and the stationary distribution of the population is relatively uniform, BNN dynamic converges faster than replicator dynamic. This is because replicator dynamic tends to drift to more popular strategies, and it is hard to exit from the initial biased point because the initially dominant strategy is relatively more preferred. However, in the opposite case when the initial distribution is relatively uniform and the stationary distribution is biased, replicator dynamic converges faster than BNN dynamic, because BNN dynamic continuously selects suboptimal strategy uniformly.

VI. User Association Algorithm

A. Theory-motivated Algorithm Development

As discussed in earlier sections, in the energy-efficient user association from the population game's perspective described as a set of differential equations, the system-wide optimum—which coincides with Nash equilibrium point—is expressed by the optimal population splitting across classes, found by the evolutionary dynamics. However, such dynamics do not directly connect to practical association algorithms, because such dynamics are described just by the portion of populations (i.e., y_i^q) over a continuous time framework, and it is still unclear how users should behave and how we should implement the dynamics in practice, e.g., which information should be exchanged, etc. In this section, we develop a practical association algorithm motivated by one of such evolutionary dynamics. To that end, we assess which dynamic is the best candidate by investigating the required information to exchange and the speed of convergence, and then finally decide to use the BR dynamic for developing a practical algorithm because of the following reasons.

First, in the replicator dynamic (see (6)), we need to emulate random encountering by letting the BSs distribute the entire users' population mass of association pattern $\{y_i^q, i \in \mathcal{S}^q\}$ and BS load $\{\rho_i, i \in \mathcal{S}^q\}$ to each user in class q. Then, each user needs to choose its *virtual* opponent based on the distributed $\{y_i^q\}$, and locally calculate the excess payoff based on $\{F_i^q(\mathbf{y})\}$ and $\{\rho_i\}$. Second, in the BNN dynamic (see (7)), BS needs to broadcast the average payoff of the users, i.e., $\frac{1}{d^q}\sum_{i\in\mathcal{S}^q} y_i^q F_i^q(\boldsymbol{y})$, and the utilization $\{\rho_i, i \in \mathcal{S}^q\}$, based on which each user calculates its excess payoff for each strategy. By contrast, BR dynamic is simper; it can be implemented by users' being given only BSs' utilization $\{\rho_i, i \in \mathcal{S}^q\}$ because each user needs just its own marginal payoff value for each BS selection $\{F_i^q(\mathbf{y}), i \in \mathcal{S}^q\}$ which solely depends on $\{\rho_i\}$.

Thus, the BR dynamic seems more *autonomous* than the other two in the sense that users do not have to know other users' strategies, and thus much fewer amount of information exchange is needed. Moreover, as presented in Section VII-A, the BR dynamic converges (numerically) much faster than the other two dynamics. This motivates us to henceforth focus on developing our practical association algorithm on the basis of the BR dynamic.

B. Algorithm Description and Rationale

We first describe our algorithm in Algorithm 1, called **BRUTE** (Best Responding User association with Traffic Estimation) [3], and then explain its design rationale. **BRUTE** is composed of the algorithms of users and BSs, which we now elaborate as follows:

Algorithm 1: BRUTE (Best Responding User association with Traffic Estimation)

Require: (i) set of BSs $\mathcal{S}$, (ii) set of user classes $\mathcal{Q}$, (iii) set of BSs $\mathcal{S}^q$ that can associate a user in class q, and (iv) set of classes $\mathcal{Q}_i$ that can be potentially associated with BS i.

Each user u in some class q

1: **for** each "association update clock" ticks **do**

2: Calculates its marginal payoffs $(\tilde{F}_i^q : i \in \mathcal{S}^q)$ using $(\tilde{\rho}_i : i \in \mathcal{S}^q)$ (broadcast by each BS) by:

$$\tilde{F}_i^q = \frac{c_i^q}{(1-\tilde{\rho}_i)^{-\alpha} + \eta P_i(1-q_i)} \tag{10}$$

3: User u chooses and is associated with the BS $i^\star$ that provides the maximum marginal payoff, i.e.,

$$i^\star = \arg\max_{j\in\mathcal{S}^q} \tilde{F}_j^q.$$

4: **end for**

Each BS i

1: Measures $\tilde{\gamma}^q$ by calculating the average traffic rate of the class q, where $q \in \mathcal{Q}_i$.

2: **for** each association update (from users) **do**

3: Computes the load by:

$$\tilde{\rho}_i = \sum_{q\in\mathcal{Q}_i} y_i^q \frac{\tilde{\gamma}^q}{c_i^q}.$$

4: Broadcasts $\tilde{\rho}_i$ to the users in each class of $\mathcal{Q}_i$.

5: **end for**

- ***User:*** Each user u updates its association whenever "association udpate clock" ticks (Line 1). Association update clock can be implemented by various ways, which include the use of a Poisson clock with some rate or flow arrivals/departures. When the clock ticks, the user u computes the potential marginal payoffs from each of BSs that can associate u (Line 2), and chooses the BS generating the maximum marginal payoff (Line 3). This marginal payoff based BS selection is motivated by the Best Response dynamic, as explained in (8). Note that (10) is obtained by a simple re-expression of (5), where (10) can be written as $\gamma^q/(\varrho_i^q[(1-\tilde{\rho}_i)^{-\alpha} + \eta P_i(1-q_i)])$, and since γ^q is invariant to the selection of i, minimizing the numerator becomes equivalent to maximization of (5).
- ***BS:*** The main role of each BS i lies in broadcasting its load periodically to the users that can be potentially

[3] We use the tilde notation in $\tilde{\gamma}^q$, $\tilde{\rho}_i$, and $\tilde{F}_i^q$ to highlight that such values are empirically measured.

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associated with i, so that users update their associations, and thus the system converges to an optimal association configuration. To that end, each BS i maintains the traffic statistics, i.e., $\tilde{\gamma}^q$ for each class q whose users have i as an association candidate (Line 1). These traffic statistics may be obtained by traffic rate measurement. Then, for each association update (made by users), it computes the load based on the measured traffic statistics (Line 3), and broadcasts the load (Line 4).

Note that each BS i and users in class q know c_i^q a priori similarly to knowing the achievable rate in user scheduling (e.g., Proportional fair), which is possible by measuring channel gains through pilot channels and exchanging feedback messages.

We briefly present the rationale of **BRUTE**; two main issues are (i) when users' associations should be updated and (ii) how BSs' loads should be measured. In particular, note that the asynchronous association and the load estimation are occasionally updated based on an asynchronous clock. Alternatively, [9], [10] assumed that the load can be simply determined by measuring their busy times and launch users' association updates controlled by a global clock, say every T time units. However, we will see that it may lead to severe performance degradation as will be shown in Figs. 5, 6, and 7.

(a) *Asynchronous association update.* This is necessary because the synchronous association updates of multiple users (in the same class) generate so-called "ping-pong" effect, resulting in the oscillation and thus leading to divergence from the optimal splitting ratio. The ping-pong effect is also undesirable, since too frequent association changes incur the overheads to the backhaul networks and thus provide bad user QoE (Quality-of-Experience).

(b) *Load measurement based on traffic estimation.* BS loads may be obtained by directly measuring the long-term ratio of busy time. Such a busy-time based method requires the load measurement to be re-initiated and investigated for a certain duration (without changing the association states) to see the average behavior whenever there is a user association change, which results in very slow convergence. By contrast, load measurement based on estimation of traffic statistics prevents the system's convergence from being governed by busy-time measurement speed. This enables us to speed up the time-scale of association updates for fast convergence, which is highly beneficial in being more robust to the network changes.

VII. Simulation Results

This section consists of two parts. The first part presents the behavior of evolutionary dynamics to show the convergence behavior and its speed discussed in Section V. The second part shows the simulation result of **BRUTE**, where we implement an event-driven simulator using SimPy [40], publicly available in [41]. The simulation results are made based on a two-cell scenario and a real cell deployment scenario.

A. Numerical Results: Three Evolutionary Dynamics

We consider a simple cellular network topology, as shown in Fig. 2(a) consisting of two urban macro BSs within 10×10 km^2. The transmission power of BS 1 is 43dBm and BS 2 is 40dBm. The maximum operating power of BSs is 865W. SINR value for determining link capacity was calculated from the modified COST 231 Hata path loss model from IEEE 802.16m (mobile WiMAX) document [42]. Using the calculated SINR values, AMC (Adaptive Modulation and Coding) was also simulated from the mobile WiMAX standard [43], [44]. The red solid and blue dashed contours represent the AMC level separations of BS 1 and BS 2, respectively. In this two-cell scenario, we ignore the inter-cell interference. This is not entirely unrealistic, because the current cellular standard uses fractional frequency reuse (FFR) and adjacent cells use different frequency band in order to reduce inter-cell interference. Note that the shaded region depicts the potential cell boundary between BS 1 and BS 2; all users in these region receive the same data rate from two BSs, and the decision metric becomes identical (i.e., a tie occurs) when the loads of two BSs are equal. The load-balancing factor α is set to 2, i.e., delay-optimal, and the energy-delay tradeoff factor η is set to 10^{-1}. Both BSs are assumed to be energy-proportional. We assume the spatially homogeneous traffic distribution, so we henceforth denote γ^q as just γ. Homogeneous traffic distribution is just adopted for simplicity, but similar interpretations in this section can be made for other heterogeneous cases.

Figs. 2(b) and 2(c) show the traces of social objective function values of three dynamics over iterations. In terms of the convergence, three dynamics show convergence to the same point, which is a NE and also the socially optimal solution, as seen in Fig. 2(b). However, the convergence speed of each dynamic is quite diverse, depending on the initial points except the BR dynamic. BR converges fastest, but that replicator and BNN dynamics show situation-dependent convergence speeds, as discussed next. Fig. 2(b) starts with heavily (1% vs. 99%) biased association in the shaded region in Fig. 2(a). The socially optimal association ratio from our calculations is about 18%–82% split in the boundary region, and deterministic association to the closest BS in other regions. Fig. 2(b) shows that replicator dynamic deviates from the initial point much more slowly than other dynamics. This occurs because replicator dynamic tends to select more "popular" BSs, i.e., the BSs with bigger portion of users have selected. Fig. 2(c) shows the opposite scenario, where the initial point in shaded region is set 20%–80%, which is near-optimal, and one of non-shaded region is set as 50%–50% distribution. In non-shaded region, the optimal strategy is that all users simply select the BS that gives the higher data rate. Starting from 50%–50% distribution, replicator dynamic converges fast to the optimal distribution, whereas BNN dynamic does not. The intuition is as follows; while in replicator dynamic users tend to pick more popular strategies, in BNN dynamic users choose one strategy at random and compare its payoff with average payoff at each selection instant. In BNN dynamic, users are more likely to make choose suboptimal decisions, compared to

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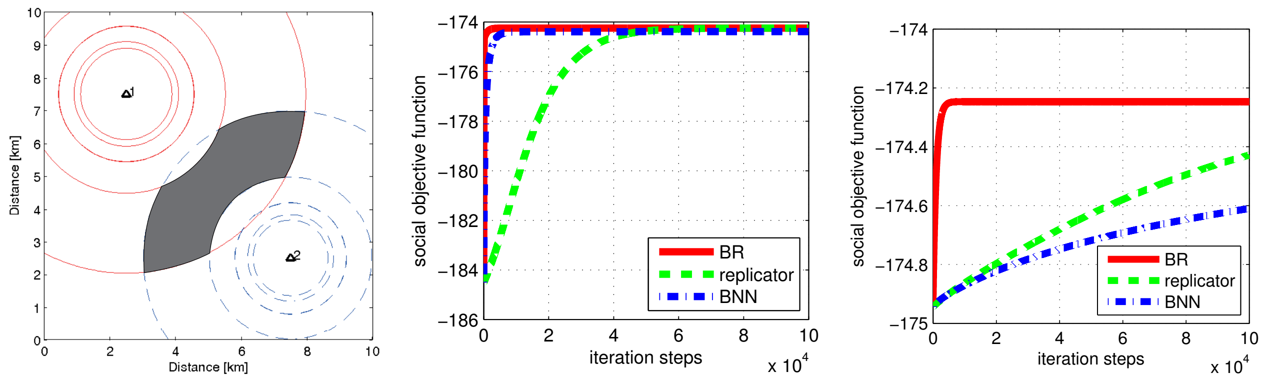

(a) Two cell scenario. The red solid and blue dashed lines represent the coverages of BSs 1 and 2, respectively. The shaded region represents the tie region between BSs 1 and 2.

(b) Case I: The initial association ratio in the shaded tie region in Fig. 2(a) is 1%-99%.

(c) Case II: The initial association ratio in each of the non-shaded regions in Fig. 2(a) is 50%-50%, and the initial association ratio in the shaded tie region is 20%-80%.

Fig. 2. Numerical results for a two-cell scenario

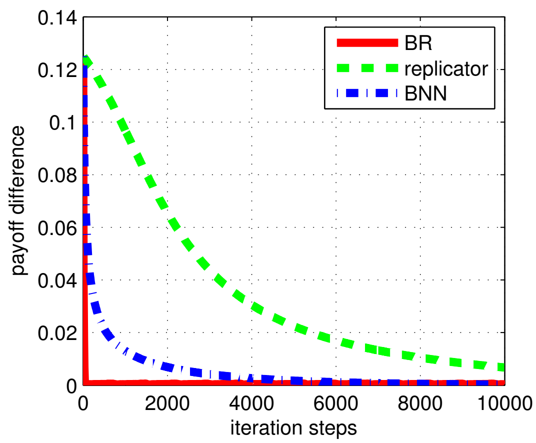

Fig. 3. The payoff difference of two strategies in shaded region in Fig. 2(a) when running scenario in Fig. 2(b).

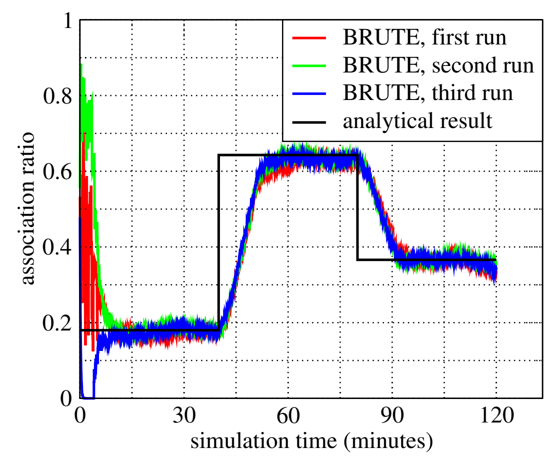

Fig. 4. The ratio of users on cell boundary which selects each BS. The ratio close to 1 indicates the more portion of boundary users are associated to BS 1. The solid black line indicates the analytically optimal association ratio. The flow arrival rate of the users close to BS 2 is changed to 1.5×10^{-2} and 1.2×10^{-2} flows/sec in 40 minute and 80 minute epoch, respectively.

replicator dynamic, resulting in drastically slow convergence. Note again that BR dynamic is observed that be relatively independent from the initial point and dominates other two dynamics in terms of the convergence speed.

Fig. 3 demonstrates that the convergence point is Nash equilibrium. From the definition of Nash equilibrium, all strategies that are contained in best response correspondence has to give equal payoff. In the scenario of Fig. 2(b), the association ratio of shaded region is initially 50%–50%, thus the payoff of two strategies have difference. The payoff difference converges to zero with the progression of the dynamics, which implies that the system state converges to Nash equilibrium.

B. Performance Evaluation of **BRUTE**

In this section, we evaluate the performance of **BRUTE**, where we first test a simple two-cell scenario to discuss the basic features such as convergence to an optimal point and robustness to traffic profile changes. In our simulation results, we implemented **BRUTE** and configured the tested networks using a discrete event simulation, SimPy [40] that implements discrete, asynchronous flow arrivals/departures and their queueing behaviors as well in each BS. The source code of our simulation is available in [41].

A Two-cell Scenario

We start by a two-cell scenario which is simple, yet showing the key features of our algorithm, i.e., how the users in the cell boundary select their associating BSs and converge. The simulation environment used here is identical to that in Section VII-A, where we have two BSs with transmitting powers 43 dBm and 40 dBm each, on 10×10 km^2 region, as shown in Fig. 2(a) (however, note again that the association algorithm is implemented by using our real algorithm **BRUTE** and flow dynamics are generated as discrete random processes so as to observe their real queueing behaviors in our simulator). We scatter 10,000 mobile users in the region, each of which generates flows according to a Poisson process with rate 10^{-2} flows/sec, with exponentially distributed file size with average 100 kbits. In addition, we setup traffic environment to change dynamically over time to see whether our proposed **BRUTE** is adapting well into dynamic situation.

Fig. 4 plots the association portion changes of the users in the cell boundary (i.e., the shaded region of Fig. 2(a)) over time, compared to the numerically computed optimal ratio (the solid black line). As mentioned in Section V, the optimal

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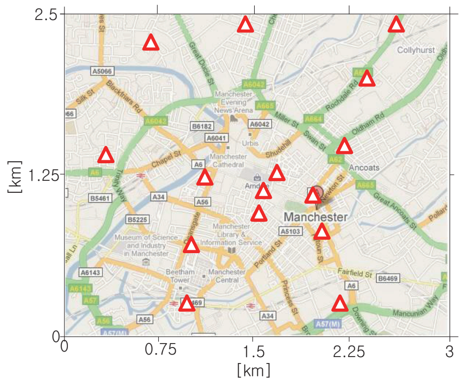
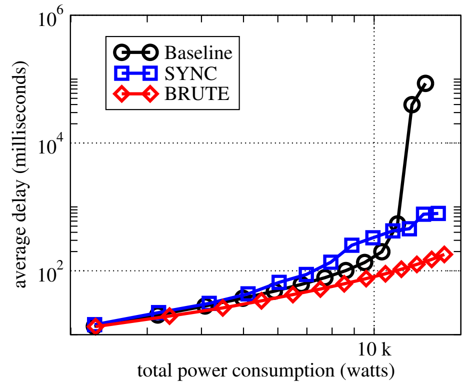
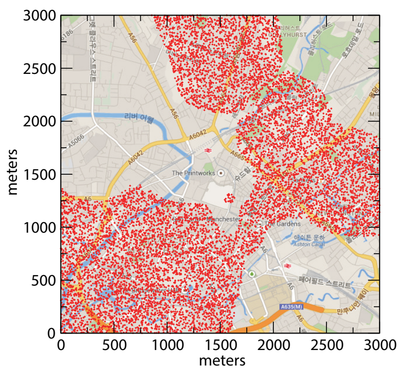


(a) A 3 km × 3 km real macro BS map (15 BSs) in Manchester, United Kingdom for our simulation.

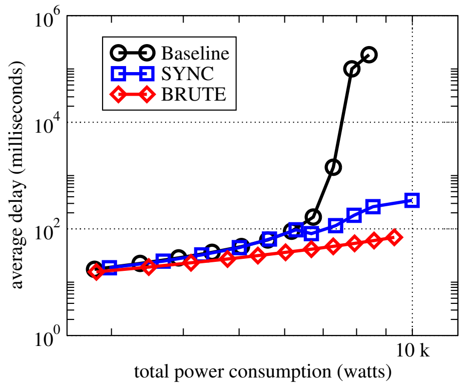


(b) The delay-energy performance in a homogeneous traffic scenario when the flow arrival rate is fixed and the average flow size varies from 100 to 580 kbits with 40 kbits step.

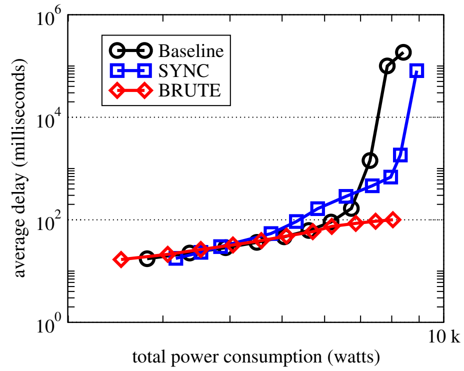
Fig. 5. A real BS map in UK and the simulation results in homogeneous user setup. 143 user classes were formulated in homogeneous user distribution.



(a) An extremely heterogeneous user setup, where the users are placed only next to the BSs indexed with even numbers.

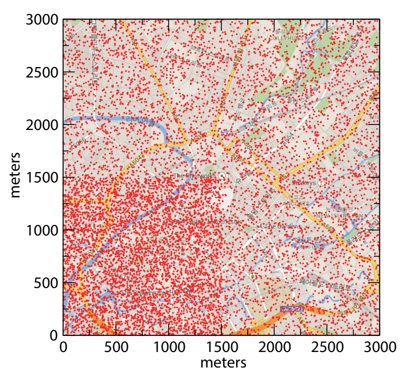


(b) The delay-energy performance in the heterogeneous setup in Fig. 6(a), when the flow arrival is fixed and the average flow size varies from 100 to 300 kbits with 20 kbits step.

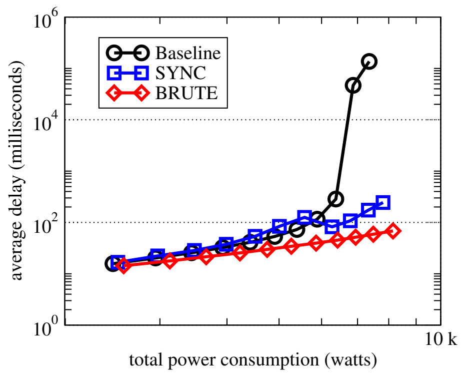


(c) The delay-energy performance with the same setup as Fig. 6(b) except for $\eta = 10$.

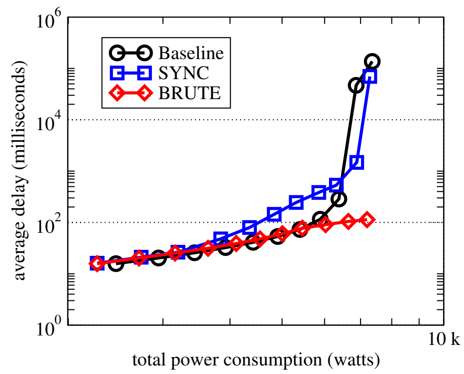
Fig. 6. Simulation results in extremely heterogeneous distributed scenario. In this case, only 87 user classes are created because of extremely heterogeneous user distribution.



(a) Another heterogeneous user setup, where 5,000 users are distributed within 1.5 × 1.5 km and rest 5,000 users are distributed elsewhere.



(b) The delay-energy performance in the heterogeneous setup in Fig. 7(a), when the flow arrival is fixed and the average flow size varies from 100 to 300 kbits with 20 kbits step.



(c) The delay-energy performance with the same setup as Fig. 7(b) except for $\eta = 10$.

Fig. 7. Simulation results in another heterogeneous distributed scenario. The number of classes is as same as that in homogeneous scenario.

association ratio shows splitting only in a shaded tie boundary region, and thus we focus only on the association ratio of that region. We observe that **BRUTE** shows fast and stable convergence behaviors and well-adapting to dynamically changing traffic environments. When the traffic intensity around BS 2 increases, the boundary users tend to associate more with BS 1,

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and **BRUTE** adapts such situation. The association ratio in the boundary region nearly converges to the optimal ratio in about ten minutes, thus our algorithm can be said that it effectively realizes the behavior of evolutionary dynamics. Moreover, our simulation runs multiple times with initial strategies randomly selected, in order to verify the global convergence of **BRUTE**. The figure shows that the solution trajectory converges to the same point regardless of the initial point.

A Real BS Map

We also perform simulations in a realistic multi-cell scenario, for which we take a 3 km $\times$ 3 km BS map investigated as of 2012 in Manchester, United Kingdom, as shown in Fig. 5(a). The location of BSs and the operation parameters of each BS are brought from Sitefinder [45]. In this environment, there are 15 macro BSs in the downtown Manchester. We scatter 10,000 active mobile users in this region randomly, the flow generation process of each user is assumed to be Poisson with 10^{-2} flows/sec arrival rate. The average flow size differs by simulations in order to generate various scenarios with different traffic intensities, i.e., different value of γ's. We assumed there are inter-cell interference between each BSs in this simulation environment and take the interference into account to calculate the data rate based on SINR. The simulation duration is 3 hours in all scenarios. The transmit power and maximum operating power of each BS are set ranged from 40 to 59 dBm and from 62 to 65 dBm, respectively. All these parameters are brought from [45]. The BSs were assumed to be energy-proportional, i.e., $q_i = 0$ for all i. We compare **BRUTE** to other two conventional algorithms as described in what follows: (i) **Baseline** and (ii) **SYNC.** First, **Baseline** is a rate-based scheme where a user is associated to the BS providing the largest data rate, and uniformly select BSs whenever there is a tie. Second, **SYNC** is the algorithm proposed in [10], where it behaves similarly to **BRUTE** except that it does not implement asynchronous association clock.

Fig. 5(b) shows the simulation results when the users are uniformly placed at random with a homogeneous traffic setup and the delay-power tradeoff constant η was set to 0.1. We gradually vary the traffic intensity ranging from 1.0 to 5.8 kbps per user, and plot the average flow delay and the corresponding total power consumption in the y-axis and x-axis, respectively. As expected, as the traffic intensity increases, both power consumption and delay increases. In the light loads, smartness of association have marginal effect, but as the load becomes higher, the impact of load-aware association becomes more important. Fig. 5(b) shows that when the traffic intensity exceeds 4.6 kbps, the delay of **Baseline** significantly grows, whereas **BRUTE** still maintains reasonably low delays. **SYNC** also considers the BS loads in association of mobiles, but **SYNC** does not support splitting within classes. Note that **Baseline** does random tie breaking. This is the main reason why **Baseline** outperforms **SYNC** in some scenarios. In the heavy traffic scenario, the delay performance of **Baseline** is worst since it does not consider BS load at all. Although **SYNC** does not support splitting, the time portion of associating with each BS can be split, therefore showing improved performance compared to **Baseline**.

Moreover, we generate two (spatially) heterogeneous scenarios, where users are distributed as in Fig. 6(a) and 7(a). In this setup, the BS utilization would be extremely unbalanced without a proper load-aware association. Fig. 6(b) and 7(b) shows the delay and energy performance with growing traffic intensities in this scenario with $\eta = 0.1$. The average delay of **Baseline** increases more rapidly than in the homogeneous traffic scenario, i.e., from the intensity 2.2 kbps, whereas **BRUTE** balances the load properly in the heterogeneous case.

Fig. 6(c) and 7(c) shows the delay and energy performance in the heterogeneous traffic scenario, with a different tradeoff parameter, i.e., $\eta = 10$. The results in Figs. 5 and 6 show that the BSs consume more power in **BRUTE** than in **Baseline** for the same traffic intensities. We overcome these results by increasing the value of η. According to Fig. 6(c) and 7(c), the power consumption of **BRUTE** (the horizontal position of the curve) decreases where the average delay has been increased compared to Fig. 6(b) and 7(b). This implies that there exists a tradeoff between average delay and energy consumption and this tradeoff can be adjusted by determining the constant η. Therefore we can say that our proposed **BRUTE** not only dramatically increases performance in the heavy-traffic scenario, but also provides the additional degree of freedom in terms of parameter setting, when we take account of energy saving as well as improving delay performance.

VIII. Conclusion and Discussions

In this paper, we have studied an energy-efficient BS association problem from a population game perspective. Our study has revealed that various evolutionary dynamics based on a population game converge to the socially optimal point through appropriate association pricing. We have also investigated on the design rationales and a practical implementation of the BR-inspired association algorithm, named **BRUTE**, and have evaluated the performance of our proposed algorithm in various simulation environments.

We conclude our paper by providing discussions related to this work. In [9], [10], the authors proposed a distributed association algorithm that solves the social optimization problem in (2) from an *optimization-theoretic* perspective. Thus, our work is of independent interest, since a different tool such as population games is utilized toward energy-efficient user association. However, as will be described in what follows, there exists critical differences from [9], [10] in other technical aspects.

(a) Formation of user classes. The authors of [9], [10] assumed an information theoretic capacity (i.e., that by Shannon) for the achievable, potential data rate. Consequently, the data rate is a continuous function of the distance between BS and user, and the vector of data rates have *continuous* real values. This makes an infinite number of user classes since the user classes are characterized with the vector of data rates from each BS. Under this somewhat impractical assumption, the authors of [9], [10] prove that the optimal user association is characterized in a deterministic manner. However, this is not practical since AMC (Adaptive Modulation and Coding) on data rate is applied, and thus the data rates are discrete in

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practice. Then, the number of classes is not infinite, implying that that the deterministic association of each user class may lead to load imbalance among BSs. The optimal association should be split within the same class in some cases such as the shaded tie region in Fig. 2(a). Using a population game-theoretic approach we show that the algorithm goes to the optimal point when there are only a finite number of user classes. This kind of behavior cannot be achieved in the deterministic algorithm in [9], [10].

(b) Time-scale separation assumption. It was assumed in [9], [10] that the time scale separation between flow arrival/departure process and load broadcasting implying that the flow arrival/departure dynamics should be much faster than the load broadcasting interval. This assumption implicitly requires an algorithm to use a long interval of load broadcasting. The algorithm designed under this assumption was optimal in [9], [10]. However in our setting where the optimal association should be splitted, the matter of asynchronicity is the key design of the algorithm since the users in a same class have to be able to respond differently. In the design of **BRUTE**, we eliminated the assumption of time scale separation with traffic estimation scheme, where load broadcasting interval does not have to be long anymore, and the users can response immediately to instantly broadcasted load information of the BSs.

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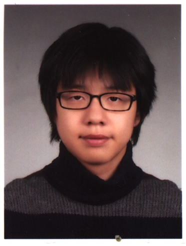

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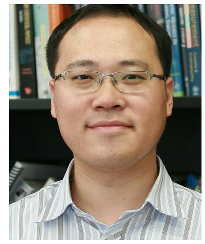

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