

# Economics of Fog Computing: Interplay among Infrastructure and Service Providers, Users, and Edge Resource Owners

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**Abstract**—Fog computing is a paradigm which brings computing, storage, and networking closer to end users and end devices for better service provisioning. One of the crucial factors in the success of fog computing is on how to incentivize the individual users' edge resources and provide them to end users such that fog computing is economically beneficial to all involved economic players. In this paper, we model and analyze a market of fog computing, from which we aim at drawing practical implications to uncover how the fog computing market should operate. To this end, we conduct an economic analysis of such user-oriented fog computing by modeling a market consisting of ISP (Infrastructure and Service Provider), SUs (end Service Users), and EROs (Edge Resource Owners) as a non-cooperative game. In this market, ISP, which provides a platform for fog computing, behaves as a mediator or a broker which leases EROs' edge resources and provides various services to SUs. In our model, a two-stage dynamic game is used where in each stage, there exists a dynamic game, one for between ISP and EROs and another for between ISP and SUs, to model the market more practically. Despite this complex game structure, we provide a closed-form equilibrium analysis which gives an insight on how much economic benefit is obtained by ISP, SUs, and EROs from user-oriented fog computing under what conditions, and we figure out the economic factors that have a significant impact on the success of fog computing.

**Index Terms**—Edge network, Fog computing, Game theory, Network Economics.

## 1 INTRODUCTION

Gartner predicts that about 21 billion “things” across different industries will be connected to the network by 2020 [2]. We are also witnessing a growing number of things at the edge providing and sharing compute, storage, sensing, and network resources. We expect this to become more individually-owned and managed in the future. Example applications include mobile cloud computing [3], [4], and content (e.g., sensing and video streaming) provisioning [5], [6]. This trend is often referred to as *fog computing and networking* (simply fog computing throughout this paper) and it has begun to attract much attention in the industry and the academe. This paradigm shift can be understood as following the philosophy of sharing economy in the area of computing and networking, which has already experienced a huge success in other business sectors, e.g., Uber, Lyft, Airbnb.

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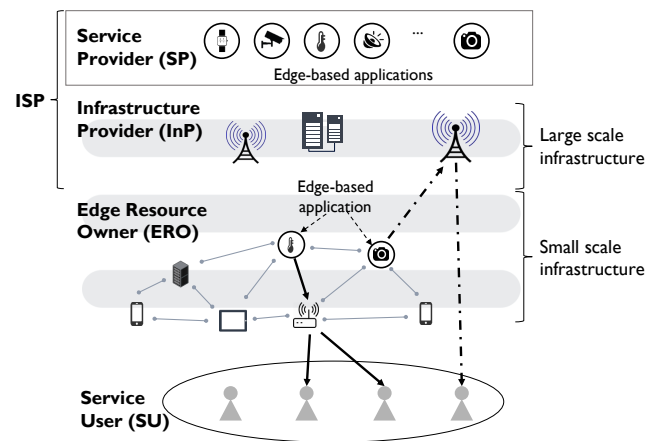


Fig. 1. Ecosystem overview of user-oriented fog computing system consisting of SP, InP, ERO, and SU. SP and InP behave as one business unit, which we call ISP (Infrastructure and Service Provider).

The market of fog computing can be explained by the economic interplay of the following four major players: (i) end Service Users (SU), (ii) Service Provider (SP), (iii) Infrastructure Provider (InP), and (iv) Edge Resource Owners (ERO), as briefly sketched in Fig. 1. SUs are the end users who are ready to enjoy edge-based applications, e.g., IoT applications. EROs are mostly individuals or companies with a small-scale communication and sensor infrastructures, who own edge resources. Especially, individual edge resource owners, like an Uber driver in the car sharing business, partially or even entirely share and sell their resources to an InP if sufficient incentive is provided. SPs create diverse edge-based applications that attract SUs as OTT (Over-The-Top) providers. SPs do not necessarily own the resources of fog clouds or edge devices but may rent them. Thus, SPs often make a contract with InPs that manage the edge resources. InPs own and manage the large-scale infrastructure of communications, sensors, and clouds but may also rely on individual EROs by running a fog network orchestration platform, e.g., [7], to expand their infrastructures. We particularly consider the case where SPs and InPs behave as one business unit, called ISP (Infrastructure and Service Provider), which is highly likely to be run by current mobile network operators (MNOs). The MNOs such as AT&T in US, and KT, SKT, LGU+ in Korea have actually started to run

such joint SP/InP business with a focus on IoT applications, e.g., [8].

As an example, we consider an augmented reality (AR) service that utilizes the fog computing platform. In this service, two types resources are needed; (i) computation resource to process video streaming, and (ii) network bandwidth resource for data transmission. AR applications run on edge devices (e.g., smartphone, smart eyeglasses) only with limited computation power and network bandwidth, yet requiring to process real-time video frames using a complex computer vision algorithm. Thus, ISP first secures both resources (a) from their own infrastructures such as central clouds or high-speed LTE networks or (b) by leasing edge resources from EROs, and then operates the AR service by assigning an appropriate amount of resources to service users. By smartly using the resources secured by either of the above ways, ISP is able to provide the AR service with high QoE (Quality of Experience) to service users. Surveillance service, real-time video analytics, and mobile big data analytics services are other examples of fog computing services [9].

In this paper, we aim to quantify how these players interact with each other and how much economic benefits do they obtain from fog computing. To this end, we model a market of fog computing where we consider a single ISP, many SUs and many EROs. by formulating an ISP-platformed two-stage “embedded” dynamic sequential game. By embedded, we mean that a sequential game is embedded at each stage. In our game formulation, we appropriately model heterogeneity of SUs and EROs in terms of willingness to pay and the QoS of shared edge resources to reflect practical factors as much as possible. To briefly explain how each stage sequential game is structured, at the first stage, ISP and EROs play a dynamic sequential game, which determines how actively EROs participate in the expansion of the fog infrastructure by being paid a certain amount of incentive. At the second stage, ISP and SUs also play a dynamic sequential game, which finally determines the ISP’s revenue and SUs’ utilities. Since ISP is the leader of the dynamic games in both stages, we call our game a *ISP-platformed* dynamic game.

Under the aforementioned market model, we conduct analytical studies to quantify which factors have how much impact on the fog ecosystem under what conditions. Despite a significantly complex game structure mainly due to their embeddings, we successfully provide a closed-form of the prices, incentives, and the resulting economic benefits (e.g., revenues and utilities) at equilibrium. To obtain more practical messages and quantify the economic gain of fog computing, our numerical results use the cost of ISP and EROs following the present price plans for cellular network and fog computing. Under this setting of costs, we draw the following useful messages: (a) ISP can increase the revenue by up to 33% by adopting the fog computing service, compared to the case where ISP runs business without collaborating with EROs, (b) the per-SU utility also increases up to 30% and that of ERO also increases. (c) Moreover, the usage of the large-scale infrastructure owned by ISP is significantly reduced due to the offloading on the EROs’ resources. (d) We also show that under any condition, fog computing generates positive benefit to all players and this benefit grows as the QoS offered by edge resource increases. See Section 4 for more implications from our analysis.

## 1.1 Related Work and Organization

Fog computing, which brings computing, storage, and networking closer to end users for better QoS, is being actively discussed

in three major groups, i.e., Cloudlet [10], MEC [11] and Open Fog Consortium [12]. In the literature, there are some researches which propose the design and implementation of fog computing, e.g., [7], [13]–[17]. The authors in [7] design a distributed operating system for fog computing system, which plays a role as a platform to manage the services and resources at the network edge, and [13] focuses on the coordinated management of fog and cloud computing systems. The work in [14] considers the framework for secure data storage and retrieval in fog computing, and [15] supports the heterogeneity of fog nodes. [17] proposes and implements edge computing platform and deploys a service on the platform. [16]–[18] are stressing to allow third-parties to create new types of services by exploiting individual resources at edge, which is the role of the service provider. The authors in [19] point out that due to the limited resources of InPs, it is crucial to provide a mechanism to incentivize the EROs for extending InP’s resources. One of the example ecosystems among EROs, SUs, and ISPs is discussed in [7], as modeled in this paper. Related to this goal, there are an array of prior works in the area of User Centric Network (UCN).

In UCN area, two categories of research are studied: *autonomous* and *network-assisted* UCN. In autonomous UCNs such as OpenGarden [20], no platform provider is involved and EROs autonomously form a network and share their resources with other EROs or users based on a pre-defined incentive mechanism. Thus, only the interaction between EROs and SUs is required, as studied in [21]–[23]. The authors in [21], [23] study an incentive mechanism and a pricing rule made by the NBS (Nash Bargaining Solution)-based resource sharing rule. The work in [22] studies incentive mechanism with coalitional game theory. However, since autonomous UCNs are their main interest, they focus on the interaction only between users without considering ISP.

From the perspective of modeling and analysis, our work is relevant with what has been studied in the area of network-assisted UCNs [24]–[30], where incentivization is usually led by ISPs. Karma [31] and FON [32] are the commercialized services of this form. In [24], [26], a two-stage Stackelberg game is modeled between an ISP and EROs where the ISP is the leader and the EROs are the followers. In [24], the authors focus on the interaction between the ISP and the hosts which operate as mobile WiFi hotspots to provide Internet connectivity service, where SUs are modeled in a highly abstract manner. The authors in [28] analyze the behavior of SUs and the ISP which operates a crowdsourced wireless community network. In [26], [27], an optimal incentive is studied, where there are *two competitive or cooperative* ISPs in the market. In [26], they model the utility of users, which is identical for both EROs and SUs, with focus only on maximizing users’ total utilities rather than taking into account individual ones. There exist related works in other domains which have similar structure to what has been modeled in this paper for fog computing market. In crowdsourcing, in [33], [34], the authors design auction-based incentive mechanism, and in [35], [36] an incentive mechanism is designed and analyzed based on game theory. The authors in [37] study the smart grid by modeling the market as a Stackelberg game.

**Difference from prior work.** The key difference of this paper from prior work lies in explicitly considering the interaction of all three players in one market with the context of user-oriented fog computing. An ISP is placed as a mediator between EROs and SUs by treating them as the same level of economic players,

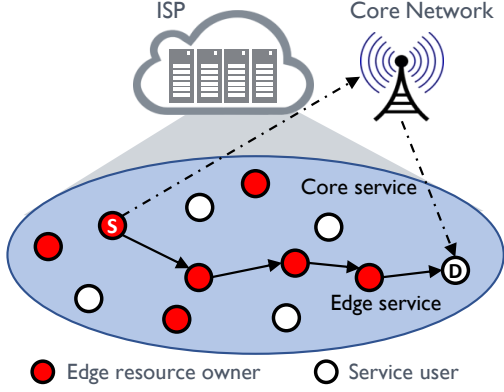


Fig. 2. Two types of services in Fog Computing System.

thereby two sequential games (one between ISP and another between ISP and SUs) are necessary to be embedded in a larger dynamic game. The work in [30] is close to ours in terms of considering all three players, but it focuses more on the stable revenue sharing based on Shapley Value in a strategical cooperation among cellular resource owners (ISP and EROs) where the cooperation's revenue is determined by the interaction with clients. However, this paper considers an ISP-platformed competitive interactions among three players (ISP, EROs, and SUs) for modeling major features of ISP in fog computing, which are (i) the competition with SUs to decide the price and (ii) the competition with EROs to determine the incentive. To the best of our knowledge, this paper is the first that studies the interactions of all three players in fog computing market which explicitly models the impact of ERO's participation on the quality of services to SUs, which is a major feature inherent to user-oriented fog computing market.

**Organization.** The rest of this paper is organized as follows: In Section 2, we first describe the system model of fog computing, followed by the game formulation among ISP, SPs, and EROs. In Section 3, we present the analysis on fog computing market which shows the equilibrium behavior of the players via a rigorous mathematical analysis and provide the numerical evaluation results in Section 4. Finally, we conclude in Section 5. Appendix includes the detail of the mathematical proofs.

## 2 MODEL AND GAME FORMULATION

### 2.1 System Model

**ISP, SUs, and EROs.** We consider a single ISP, which plays the role of both InP and SP as mentioned earlier. ISP leases the edge resources from EROs and provides a service to SUs. We assume that there are  $N$  number of SUs, and  $bN$  number of EROs, where  $b > 0$ . SUs pay the service fee to ISP, when they subscribe to the service from the ISP, and EROs decide to share their edge resource, contributing to an expansion of ISP's infrastructure, if incentives are appropriately provided.

**Services.** The ISP provides the fog computing service with two types: (i) core service and (ii) edge service. As shown in Fig. 2, the core service corresponds to the case where SUs are allowed to use a cellular mobile Internet service (e.g., LTE) and computing/storage resources in some central cloud, whereas the edge service refers to the one that SUs use only nearby edge computing/sensing/connectivity/storage resources through the network formed by edge devices. Thus, SUs can choose between core and

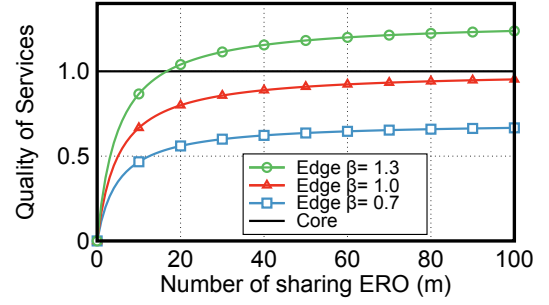


Fig. 3. Graph comparing the edge's and core's qualities for varying number of sharing-EROs and for different values of  $\beta = 0.7, 1.0, 1.3$ .

edge or none of them. The ISP sets the service prices and if SUs subscribe the service, they pay the service fee to ISP. We denote by  $p_c$  and  $p_e$  the prices of core and edge services, respectively. To provide the edge service, the ISP needs to lease the edge resources from EROs by paying them some incentive  $q$ , depending on which each ERO decides to share its resources.

**Quality of core and edge services.** We let  $\alpha_c$  denote the quality of core service. For mathematical tractability, we assume that it is homogeneous across applications. Thus, when SUs subscribe to core service, the utility of SUs purely depends on  $\alpha_c$ . We let  $\alpha_e$  be the quality of edge service and model it as:

$$\alpha_e = f(m) \quad (1)$$

where  $m$  is the number of EROs which share the resource, and  $f: \mathbb{R} \rightarrow \mathbb{R}$  is a twice differentiable, strictly increasing, and concave function, i.e.,  $f'(m) > 0$  and  $f''(m) < 0$  for all  $m$ . The quality of edge service depends on how many resources ISP has. We assume that once EROs decide to share their resources, they share the same amount of resource (i.e., unit resource). Thus, the quality of edge service depends only on  $m$ , the number of EROs which share the resource. It is clear that as the number of EROs sharing their resources increases, the quality of edge grows and we model  $\alpha_e$  to be strictly concave in  $m$  to reflect the effect of diminishing returns.

We present an example form of  $f$  which is simple but parameterizes the features of fog computing environment, as follows:

$$\alpha_e(m) = \beta \alpha_c \left( 1 - \left( a \cdot \frac{m}{N} + 1 \right)^{-1} \right). \quad (2)$$

In this model, as the portion of EROs sharing their resources relative to the total number of SUs,  $\frac{m}{N}$ , increases, the quality of edge grows, whose aggressiveness is modeled by a constant  $a$ . The maximum quality of edge service is  $\beta \alpha_c$  when the number of EROs sharing their resources is very large, i.e.,  $m \rightarrow \infty$ , where  $\beta$  quantifies the fundamental difference in QoS between core and edge services. Fig. 3 illustrates the shape of QoS of edge services for varying number of sharing-EROs  $m$  for different values of  $\beta$ . Note that the value of  $m$  is not fixed a priori, but determined by the game among all the players where our interest is in the one at the equilibrium. We will provide the analysis based on (2) as well, which gives us more analytically interpretable result.

### 2.2 Market Model: Game Formulation

We aim at understanding how ISP, SUs and EROs interplay for providing and consuming edge services in fog computing market,

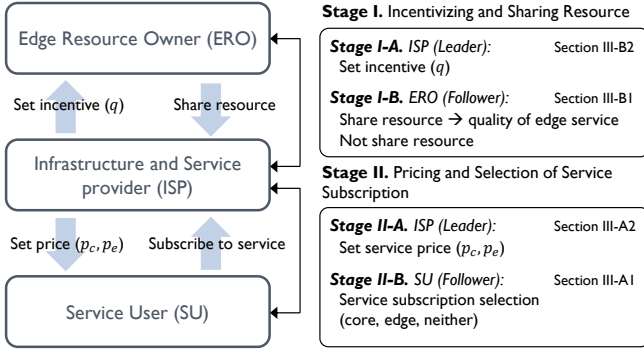


Fig. 4. Game formulation: ISP-plated two-stage dynamic game.

and analyzing their strategic behaviors to maximize their selfish objective. To this end, we model a fog computing market by a type of non-cooperative game, called ISP-plated two-stage embedded dynamic game. To help the readers, we briefly provide an overview of the game structure using Fig. 4 and formally describe it after presenting the pay-off functions of ISP, ERO, and SU.

As summarized in Fig. 4, two stages are embedded in our game. At Stage I, ISP initially leads a sequential game to collect the resources from EROs. ISP sets the incentive and EROs determine whether they share their resource or not depending on the incentive. Subsequently, ISP leads another game to operate two types of services to SUs at Stage II, where ISP sets the service prices and SUs select the service subscription according to the prices. At the equilibrium of our game, ISP determines its pricing strategies  $p_e, p_c, q$  to maximize its revenue  $\pi$  (see (3)), while EROs (or SUs) individually decide their own strategy  $y \in \mathcal{S}_{\text{ERO}}$  (or  $x \in \mathcal{S}_{\text{SU}}$ ) to maximize individual utility  $u^{\text{ERO}}$  (or  $u^{\text{SU}}$ ) where  $\mathcal{S}_{\text{ERO}}$  (or  $\mathcal{S}_{\text{SU}}$ ) denotes the strategy set of EROs (or SUs).

We now present the pay-off functions and the strategies of three players in what follows:

**Utility of ERO.** An ERO's utility is determined by various factors such as the resource sharing cost, incentive to share, and its willingness to share. To model this, we consider the following utility function:

$$u^{\text{ERO}}(q, y; \theta) = \begin{cases} q - \theta q_0, & \text{if } y = 's', \\ 0 & \text{if } y = 'n', \end{cases}$$

where  $q_0$  denotes ERO's cost for resource sharing and  $\theta$  represents the willingness to share of an ERO. Each ERO selects its strategy  $y \in \mathcal{S}_{\text{ERO}}$  among 's' and 'n' which correspond to *sharing* and *non-sharing*, respectively. To model ERO's heterogeneity, they are assumed to have different willingness to pay  $\theta$ , where as popularly modeled in e.g., [38],  $\theta$  is a uniformly random value over the interval  $[0, 1]$

For example, for a given cost  $q_0$ , an ERO with a smaller  $\theta$  has less sensitivity to cost for sharing (or more willingness to share) its resource than the one with a larger  $\theta$ .

**Utility of SU.** An SU's utility would be affected by various factors, of which we focus on the following primary factors: service fees  $(p_e, p_c)$  and QoSes  $(\alpha_e, \alpha_c)$  of *core* and *edge* services. To model

this, we consider the following utility function:

$$u^{\text{SU}}(p_e, p_c, x; \gamma) = \begin{cases} \gamma \alpha_e - p_e, & \text{if } x = 'e', \\ \gamma \alpha_c - p_c, & \text{if } x = 'c', \\ 0, & \text{if } x = 'n', \end{cases}$$

where  $x \in \mathcal{S}_{\text{SU}}$  is the strategy of SU, representing which service the SU subscribes to among 'c', 'e' and 'n', each of which corresponds to *core*, *edge* or no-subscription, respectively. The value of  $\gamma$  is the willingness to pay, again assumed to be uniformly random over  $[0, 1]$ . For a given QoS, a SU with higher  $\gamma$  has more willingness to pay than the one with smaller  $\gamma$ .

**Revenue of ISP.** The revenue of ISP consists of an income from providing the services of *core* and *edge* and an expenditure on operating and managing *core* resources and leasing *edge* resources. Then, the net-revenue of ISP is given by:

$$\pi(\cdot) = p_c n_c + p_e n_e - p_0 n_c - m q, \quad (3)$$

where we denote by  $p_0$  the constant cost for providing *core* service, and  $n_c$  and  $n_e$  the number of SUs who subscribe to the *core* and *edge* services, respectively. In (3), the first and second terms are the income from providing *core* and *edge* services, respectively. The third term is the cost for providing *core* service and the last term is the cost for paying EROs incentive to provide *edge* service. When we express (3) in more explicit relation to SUs' and EROs' strategies and willingness-to-pay and willingness-to-share, we get:

$$\begin{aligned} \pi(\cdot) = N \int_0^1 & \left\{ (p_c - p_0) \cdot \mathbf{1}_{\{\max(0, u_e(\gamma)) \leq u_c(\gamma)\}} \right. \\ & \left. + p_e \cdot \mathbf{1}_{\{\max(0, u_c(\gamma)) \leq u_e(\gamma)\}} \right\} d\gamma \\ & - b N \int_0^1 q \cdot \mathbf{1}_{\{\theta < \theta_0\}} d\theta, \end{aligned} \quad (4)$$

where  $\theta_0 = q/q_0$ . Recall that  $u_c(\gamma)$  and  $u_e(\gamma)$  are the utilities of SU with willingness-to-pay  $\gamma$ , when it subscribes to *core* and *edge* services, respectively. ISP gains from the SUs with  $\gamma$  which satisfies  $\max(0, u_e(\gamma)) \leq u_c(\gamma)$  as well as does by *edge* from SUs with  $\gamma$  which satisfies  $\max(0, u_c(\gamma)) \leq u_e(\gamma)$ . Similarly, ISP spends on leasing *edge* resources from the EROs who is willing to share with  $\theta < \theta_0$ , where  $\theta_0$  is the threshold of willingness-to-share below which an ERO does not share its resource, i.e.,  $q - \theta_0 q_0 = 0$ .

We now describe the ISP-plated two-stage dynamic game.

## ISP-plated Two-stage Dynamic Game

### Stage I: Incentivizing EROs and resource sharing of ERO.

ISP first sets the incentive  $q$  as a leader, then each ERO with willingness-to-share  $\theta$  selects its service among  $\mathcal{S}_{\text{ERO}} \triangleq \{s, n\}$ , where 's' and 'n' correspond to *sharing* and *non-sharing*, respectively.

$$\text{Stage I-A. ISP (Leader): } q^* = \arg \max_{q \in [0, 1]} \pi(q, y, p_c, p_e, x),$$

$$\text{Stage I-B. ERO (Follower): } y^*(\theta) = \arg \max_{y \in \mathcal{S}_{\text{ERO}}} u^{\text{ERO}}(q, y; \theta).$$

### Stage II: Pricing for SUs and service subscription of SUs.

ISP first decides the service prices  $p_c$  and  $p_e$ , then each SU with willingness-to-pay  $\gamma$  chooses which service to subscribe to out of

$S_{SU} \triangleq \{c, e, n\}$ , where we use the ‘c’, ‘e’ and ‘n’ to refer to an SU’s selection of **core**, **edge** or no-subscription.

*Stage II-A. ISP (Leader):*  $(p_c^*, p_e^*) = \arg \max_{(p_c, p_e)} \pi(q, y, p_c, p_e, x)$ ,

*Stage II-B. SU (Follower):*  $x^*(\gamma) = \arg \max_{x \in S_{SU}} u^{SU}(p_c, p_e, x; \gamma)$ .

### 3 FOG-COMPUTING MARKET ANALYSIS: SERVICE OPERATION AND RESOURCE AGGREGATION

In this section, we provide the equilibrium analysis for the ISP-platformed two-stage dynamic game described in the previous section. We adopt the classical backward induction to find the subgame perfect equilibrium of our sequential game. Our sequential analysis in turn gives us the answers on how ISP operates **core** and **edge** services and how SUs select the services in Stage II, as well as how ISP collects the edge resources from EROs in Stage I.

To find the equilibrium using the backward induction, we start the equilibrium analysis of the competitive market between ISP and SUs in Stage II. We investigate the service selection of SUs and the pricing of ISP in Section 3.1.1 and 3.1.2, respectively. Using the result in Stage II, we present the analysis of market between ISP and EROs in Stage I. Section 3.2.1 provides the behavior of EROs to share their resource, and Section 3.2.2 provides ISP’s selection of incentive to lease resource from EROs. In Section 3.3, we present the strategies of all players at the equilibrium and their resulting economic benefits.

#### 3.1 Interaction between ISP and SUs in Stage II: Pricing and Selection of Service Subscription

We focus on the game between ISP and SUs in Stage II, where ISP operates **core** and **edge** services by setting the service fees,  $p_c$  and  $p_e$ , and each SU determines the service for subscription. Assuming that the ISP successfully leases edge resources by giving incentive  $q$  to EROs in Stage I, then the QoS of **edge** service is determined, so that SUs make decisions based on the given price and QoS. Stage II consists of two substages where the Stage II-A is the decision on ISP’s prices  $p_c$  and  $p_e$ , and the Stage II-B is the decision on the services that SUs subscribe to. We first present the analysis of Stage II-B, followed by that of Stage II-A.

##### 3.1.1 (Stage II-B) SU’s Selection of Strategy

In this substage, heterogeneous SUs try to maximize their own utilities by selecting a service out of **core**, **edge**, and none, depending on the service fees, QoSes, and willingness to pay. Proposition 3.1 states which service an SU selects when the service fees  $(p_c, p_e)$  and QoSes  $(\alpha_c, \alpha_e)$  are given by ISP.

**Proposition 3.1.** *For given values of QoSes  $(\alpha_c, \alpha_e)$  and prices  $(p_c, p_e)$ , the utility-maximizing strategy of an SU with willingness to pay  $\gamma$  is given as:*

(i)  $\alpha_c \geq \alpha_e$

$$x^*(\gamma) = \begin{cases} c, & \text{if } \gamma > \max\left(\frac{p_c - p_e}{\alpha_c - \alpha_e}, \frac{p_c}{\alpha_c}\right), \\ e, & \text{if } \frac{p_e}{\alpha_e} < \gamma \leq \frac{p_c - p_e}{\alpha_c - \alpha_e}, \\ n, & \text{otherwise.} \end{cases}$$

(ii)  $\alpha_c < \alpha_e$

$$x^*(\gamma) = \begin{cases} c, & \text{if } \frac{p_c}{\alpha_c} < \gamma \leq \frac{p_e - p_c}{\alpha_e - \alpha_c}, \\ e, & \text{if } \gamma > \max\left(\frac{p_e - p_c}{\alpha_e - \alpha_c}, \frac{p_e}{\alpha_e}\right), \\ n, & \text{otherwise.} \end{cases}$$

It is intuitive that the SU with higher willingness prefers the service with higher QoS. As shown in Fig. 5(a), when **core**’s QoS is better than **edge**’s one (i.e.,  $\alpha_c \geq \alpha_e$ ), SUs who have relatively higher willingness to pay (i.e.,  $\gamma \geq \max\left(\frac{p_e - p_c}{\alpha_e - \alpha_c}, \frac{p_e}{\alpha_e}\right)$ ) subscribe to **core** service which returns higher utility, even if it is more expensive than **edge** (i.e.,  $p_c \geq p_e$ ). However, Fig. 5(b) shows that if  $p_c$  is too expensive (i.e.,  $p_c > p_e + \alpha_c - \alpha_e$ ), all SUs have higher utility when they subscribe to **edge** service. Similarly, when **edge**’s QoS is better than **core**’s one, the SU’s behavior shows an opposite tendency as shown in Figs. 5(c) and 5(d).

##### 3.1.2 (Stage II-A) ISP’s Selection of Service Prices

In Stage II-A, the ISP decides the service prices  $p_c$  and  $p_e$ . In Proposition 3.1, SU’s service subscription strategy depends on which range where the willingness to pay is included. The range of either **core** or **edge** or both services can be infeasible, depending on the conditions of service prices  $(p_c, p_e)$ . Thus, for given QoSes  $(\alpha_c, \alpha_e)$ , it is necessary to separately consider the service prices  $(p_c, p_e)$  in the following four disjoint regions, according to whether each service is feasible or not.

$$A_1 = \left\{ (p_c, p_e) \mid p_c < \min\left(\frac{\alpha_c}{\alpha_e}, 1\right) p_e + (\alpha_c - \alpha_e)^+, \right. \\ \left. p_c \geq \max\left(\frac{\alpha_c}{\alpha_e}, 1\right) p_e - (\alpha_e - \alpha_c)^+ \right\}$$

$$A_2 = \left\{ (p_c, p_e) \mid p_c \geq \min\left(\frac{\alpha_c}{\alpha_e}, 1\right) p_e + (\alpha_c - \alpha_e)^+, p_e \leq \alpha_e \right\}$$

$$A_3 = \left\{ (p_c, p_e) \mid p_c < \max\left(\frac{\alpha_c}{\alpha_e}, 1\right) p_e - (\alpha_e - \alpha_c)^+, p_c \leq \alpha_c \right\}$$

$$A_4 = \left\{ (p_c, p_e) \mid p_c > \alpha_c, p_e > \alpha_e \right\}, \quad (5)$$

where  $p_c \geq 0, p_e \geq 0$  and the partitions are presented in Fig. 6. In  $A_1$ , SUs subscribe to both **core** and **edge** services. In  $A_2$  (resp.  $A_3$ ), SUs subscribe to only **edge** (resp. **core**) service and in  $A_4$ , no SU subscribes to any service, because the utility becomes negative.

The revenue function of ISP in (3) has a different shape, which depends on which region  $(p_c, p_e)$  resides, because SUs select different strategy depending on the service prices, as described in Proposition 3.1. Let  $\pi_i$  denote the revenue function of ISP for  $(p_c, p_e) \in A_i, i = 1, 2, 3, 4$ . Then, the revenue function of ISP is given as follows:

$$\begin{aligned} \pi_1(p_c, p_e) &= \begin{cases} N(p_c - p_0)(1 - z) + Np_e \left(z - \frac{p_e}{\alpha_e}\right) - B, & \text{if } \alpha_c > \alpha_e, \\ N(p_c - p_0) \left(z - \frac{p_c}{\alpha_c}\right) + Np_e(1 - z) - B, & \text{if } \alpha_c \leq \alpha_e, \end{cases} \\ \pi_2(p_c, p_e) &= Np_e \left(1 - \frac{p_e}{\alpha_e}\right) - B, \\ \pi_3(p_c, p_e) &= N(p_c - p_0) \left(1 - \frac{p_c}{\alpha_c}\right) - B, \\ \pi_4(p_c, p_e) &= 0. \end{aligned} \quad (6)$$

where  $B = bN \int_0^1 q \cdot \mathbf{1}_{\{\theta < \theta_0\}} d\theta$  is ISP’s cost for leasing resource from EROs, and  $z = \frac{p_c - p_e}{\alpha_c - \alpha_e}$ . Now, following the ‘‘optimal’’



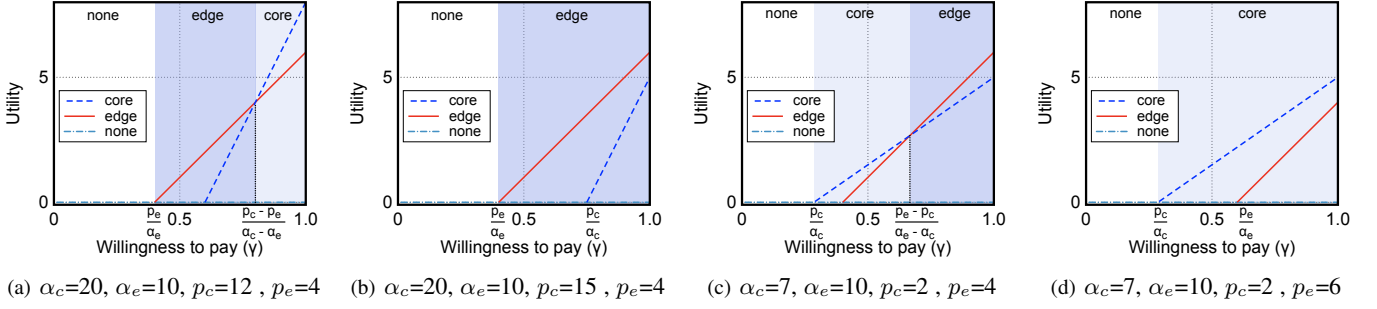


Fig. 5. SU's strategy selection with different willingness to pay ( $\gamma$ )

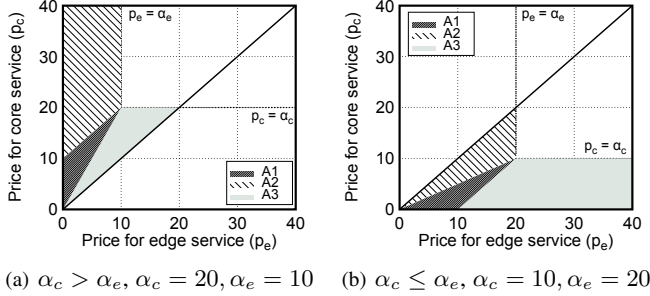


Fig. 6. Partitions of service prices depending on SUs' behavior.

decisions by SUs at Stage II-B, the ISP chooses the prices of edge and core services to maximize its revenue, as stated in Proposition 3.2.

**Proposition 3.2.** For given QoSes ( $\alpha_c, \alpha_e$ ), ISP sets the prices  $p_c^*$  and  $p_e^*$  as follows:

$$p_c^* = \frac{p_0 + \alpha_c}{2}, \quad p_e^* = \frac{\alpha_e}{2}. \quad (7)$$

The ISP is the leader in Stage II and thus it knows how SUs act in backward induction. When the QoS of edge service is given, ISP determines the price to maximize its revenue by predicting SUs' optimal action. Since the QoS of core service is independent of the amount of leased edge resource, the price of core service  $p_c$ , is not a function of the quality of edge service. Thus, by only controlling the price of edge service  $p_e$ , ISP maximizes its revenue. As the QoS of edge service grows, the selected price for edge service increases. It is natural that the better QoS of edge service and the more SUs intend to subscribe to edge service, so that increasing the price can help in increasing the revenue. We obtain the following corollary, which is based on Proposition 3.2, that characterizes the conditions of ISP's prices selection.

**Corollary 3.1.** For given QoSes ( $\alpha_c, \alpha_e$ ), the prices ( $p_c^*, p_e^*$ ) is in either  $A_1$  or  $A_2$ .

Somewhat surprisingly, with different partitions considered for separate forms of ISP's revenue as in (6), we obtain a simple form of ISP's price selection as in (7), which helps a lot in our closed-form analysis in Stage I. Corollary 3.1 enables us to study ISP's price selection under only two cases.

**Two regimes: core-preferred and edge-dominant.** We henceforth refer to the regime where ( $p_c^*, p_e^*$ ) is in  $A_1$  as R1: *core-preferred*

regime, and the regime where ( $p_c^*, p_e^*$ ) is in  $A_2$  as R2: *edge-dominant regime*. In R1, core service is preferred by SUs whose willingness to pay is relatively high and SUs with lower willingness to pay subscribe to edge service, whereas in R2, edge service dominates core service such that no SU subscribes to core service.

### 3.2 Interaction between ISP and EROs in Stage I: Incentivizing and Sharing Resource

In this subsection, we analyze the sequential subgame between the ISP and EROs in Stage I. We now aim at understanding how the decision of ISP's incentive to EROs affects the behavior of EROs. Following the backward induction, we first explain how each ERO determines whether it shares the resource or not with a given incentive  $q$  in Stage I-B, where the incentive  $q$  is determined by ISP in Stage I-A.

#### 3.2.1 (Stage I-B) ERO's Selection of Strategy

In this substage, we look into ERO's selection of strategy to share the resource. For a given incentive  $q$  by ISP, the strategy of an ERO with willingness to share  $\theta$  is given as follows:

$$y^*(\theta) = \begin{cases} s, & \text{if } \theta < \frac{q}{q_0}, \\ n, & \text{otherwise,} \end{cases} \quad (8)$$

where, as mentioned earlier, the threshold on willingness to share  $\theta_0$  is  $\theta_0 = \frac{q}{q_0}$ . When willingness to share of an ERO is assumed to be uniformly random in  $[0, 1]$ , the number of EROs who share their resources is  $\frac{bNq}{q_0}$ . Using (1), the shared edge resources' QoS becomes:

$$\alpha_e = \alpha_e(q) := f\left(\frac{bNq}{q_0}\right), \quad (9)$$

where we often use  $\alpha_e(q)$  to explicitly express its dependence on  $q$ . Recall  $f'(m) > 0, f''(m) < 0$  for all  $m$  as described in (1). Furthermore,  $\frac{bNq}{q_0}$  is a linear function of  $q$ ,  $\alpha_e'(q) > 0$  and  $\alpha_e''(q) < 0$  for all  $q \in [0, q_0]$ .

#### 3.2.2 (Stage I-A) ISP's Selection of Incentive

In Stage I-A, ISP leases edge resources from EROs by giving a suitable amount of incentive. In backward induction, as the leader of game, the ISP predicts EROs' equilibrium behaviors depending on its incentive decision. Our central interest is that in conjunction with the results in Stage I-B, how much incentive ISP should give to EROs and what is the revenue of ISP at the equilibrium of the entire game. The ISP's revenue function has different shape,

depending on in which region the decision of ISP's prices  $(p_c, p_e)$  resides. Thus, we first investigate how ISP's decision of incentive affects the decision of prices and SUs' behavior in Stage II.

From Corollary 3.1, we have mentioned that depending on ISP's choice of service prices, there are two regimes; R1: *core-preferred regime*, and R2: *edge-dominant regime*. In Proposition 3.2,  $p_e^*$  is determined by the QoS of **edge** service which is in turn controlled by the incentive  $q$ . Thus, the regime of ISP's strategy is determined by following the condition of  $q$ :

$$\text{C0} : q < \alpha_e^{-1}(\alpha_c - p_0),$$

where  $f$  is the quality of **edge** service from (1). This means that when C0 holds, the decision of ISP's prices falls in  $A_1$  and it is in the regime R1 where there exist three types of SUs subscribing to one of **core**, **edge**, or no-subscription. Otherwise, it falls in  $A_2$ , thus being in the regime R2 where no SUs subscribe to the **core** service.

The revenue of ISP in (3) is the function of the prices  $p_c$  and  $p_e$ , and the incentive  $q$ . From Proposition 3.2 in Stage II, ISP determines the prices  $(p_c^*, p_e^*)$  for given  $q$  where  $p_e^*$  is the function of  $q$ . Let  $\pi^*(q)$  denote the revenue of ISP when the ISP selects the prices which maximize the revenue. Then, we can rewrite the revenue function as follows.

$$\pi^*(q) = \begin{cases} \pi_1^*(q) & \text{if C0 holds,} \\ \pi_2^*(q) & \text{otherwise,} \end{cases} \quad (10)$$

where we abuse the notation  $\pi_i^*(q)$  as  $\pi_i(p_c^*, p_e^*)$  in (6) to stress its dependence on given  $q$ . In order to maximize the revenue, ISP sets the incentive  $q^*$  which maximizes (10), and the result is presented in following subsection.

### 3.3 Equilibrium of ISP-platformed Two-stage Dynamic Game

We now present the equilibrium strategies of ISP, SUs and EROs, by using the analysis in Stage I and II in Sections 3.2 and 3.1.

(a) *ISP's service prices to SUs and incentive to EROs.* Theorem 3.1 presents ISP's equilibrium strategies of service prices to SUs and incentive to EROs.

**Theorem 3.1** (ISP's service prices and incentive). *ISP's service prices to SUs and incentive to EROs at the equilibrium are given by:*

$$p_c^* = \frac{p_0 + \alpha_c}{2}, \quad p_e^* = \frac{\alpha_e(q^*)}{2}, \\ q^* = \arg \max_{\{\tilde{q}_1, \tilde{q}_2\}} \pi^*(q),$$

such that  $\tilde{q}_1 := \min(q_1^*, q_0)$  and  $\tilde{q}_2 := \min(q_2^*, q_0)$  where  $q_1^*$  is the unique solution of following equation:

$$8bq(\alpha_c - \alpha_e(q))^2 = p_0^2 q_0 \alpha_e'(q), \quad (11)$$

and  $q_2^*$  is the unique solution of following equation:

$$8bq = q_0 \alpha_e'(q). \quad (12)$$

Theorem 3.1 characterizes the equilibrium of ISP's strategies in our two-stage embedded dynamic game. It reveals how much the selection of incentive sequentially affects the pricing strategies at the equilibrium. To understand the impact of the ISP's decision of incentive in Theorem 3.1 more clearly, we will provide the numerical results which show the revenue of ISP for varying incentive and the equilibrium behaviors of all players in Section 4.

(b) *SUs' service subscription and EROs' resource sharing.* We now aim at understanding the equilibrium behavior of SUs and EROs in Proposition 3.3. Note that SUs and EROs are differentiated by their willingness-to-pay  $\gamma$  and willingness-to-share  $\theta$ , their equilibrium strategies are represented by their choices of subscription and resource sharing, depending on the values of  $\gamma$  and  $\theta$ .

**Proposition 3.3** (Equilibrium behaviors of SUs and EROs). *With the equilibrium prices and incentives  $p_c^*$ ,  $p_e^*$ , and  $q^*$  in Theorem 3.1, for given willingness-to-pay  $\gamma$  and willingness-to-share  $\theta$ , SUs' service subscription  $x^*(\gamma)$  and EROs' resource sharing  $y^*(\theta)$  are given in what follows:*

- *SUs' service subscription*

(R1) *Core-preferred regime (i.e., when C0 holds):*

$$x^*(\gamma) = \begin{cases} c, & \text{if } \gamma > \frac{p_c^* - p_e^*}{\alpha_c - \alpha_e(q^*)}, \\ e, & \text{if } \frac{1}{2} < \gamma \leq \frac{p_c^* - p_e^*}{\alpha_c - \alpha_e(q^*)}, \\ n, & \text{otherwise.} \end{cases}$$

(R2) *Edge-dominant regime (i.e., when C0 does not hold):*

$$x^*(\gamma) = \begin{cases} e, & \text{if } \gamma > \frac{1}{2}, \\ n, & \text{otherwise.} \end{cases}$$

- *EROs' resource sharing*

$$y^*(\theta) = \begin{cases} s, & \text{if } \theta < \frac{q^*}{q_0}, \\ n, & \text{otherwise,} \end{cases}$$

Let  $z^* = \frac{p_c^* - p_e^*}{\alpha_c - \alpha_e(q^*)}$ . Proposition 3.3 shows the equilibrium of SUs and EROs' strategies. In regime R1, SUs with  $\gamma \in [\frac{1}{2}, z^*]$  prefer to **edge** service, and SUs with  $\gamma \in [z^*, 1]$  decide to subscribe to **core** service. Whereas in regime R2, SUs with  $\gamma \in [\frac{1}{2}, 1]$  subscribe to **edge** service. The comparison with *core-only* case where there only exists **core** service, shows the impact of **edge** service. In *core-only* case, the price of **core** service is the same as  $p_c^*$  as proved in the proof of Proposition 3.2. Thus, SUs who subscribe to **edge** service in fog computing market, get the better utility compared to **core** service in *core-only* case. Since there always exist some SUs who subscribe to **edge** service, providing **edge** service always improves the average utilities of SUs. Moreover, in order to provide **edge** service, ISP leases resource from EROs by giving incentive to EROs, and this increases the utilities of EROs at the same time. Thus, when the ISP uses **edge** service, all players get the benefit in fog computing market.

**Special case: When the quality of edge service follows (2).** To make our result in Theorem 3.1 more analytically visible, we consider the specific model of the quality of **edge** service as (2) which is simple but can model various factors as described in Section 2.1. Under the above assumptions on the quality of **edge** service, we have the following result on the equilibrium:

**Proposition 3.4.** *Under the assumption that the quality of edge service follows (2), ISP's service prices to SUs and incentive to EROs at the equilibrium are given by:*

$$p_c^* = \frac{p_0 + \alpha_c}{2}, \quad p_e^* = \frac{ab\beta\alpha_c q^*}{2(abq^* + q_0)}, \\ q^* = \arg \max_{\{\tilde{q}_1, \tilde{q}_2\}} \pi^*(q),$$

such that  $\tilde{q}_1 := \min(q_1^*, q_0)$  and  $\tilde{q}_2 := \min(q_2^*, q_0)$  where  $q_1^*$  is the unique solution of following equation:

$$8q\alpha_c(ab(1-\beta)q + q_0)^2 = a\beta q_0^2 p_0^2,$$

and  $q_2^*$  is the unique solution of following equation:

$$8q(abq + q_0)^2 = a\beta\alpha_c q_0^2.$$

To interpret the result of Proposition, we consider a special, yet practically plausible case when  $\beta = 1$ ,  $b = 1$ ,  $a = 1$ , and  $\alpha_c = kp_0$  for some  $k > 0$ . The assumption of  $b = 1$  and  $a = 1$  is mainly from our convenience, yet practically meaning that the number of SUs are all potential EROs. From (2),  $a = 1$  corresponds to the case when the increasing pattern of  $\alpha_e$  is mild as the number of resource-sharing EROs increases, and  $\beta = 1$  means that the maximum value of  $\alpha_e$  equals to  $\alpha_c$ , i.e., in the best case when all EROs share their resources, there is not much difference between the qualities of **core** and **edge** services. The assumption of  $\alpha_c = kp_0$  implies that the basic cost of **core** service is linear in its quality.

Under the above assumptions on the parameters, the quality of **edge** service is  $\alpha_e(q) = \frac{kp_0q}{q+q_0}$ . When  $\frac{p_0}{8k(k-1)} < q_0$  holds and  $q^*$  meets condition C0, it is in core-preferred regime and the equilibrium is as follows:

$$q^* = \min\left(\frac{p_0}{8k}, q_0\right),$$

$$\pi^* = N \cdot \min\left(\frac{p_0^2}{64k^2q_0}, \frac{1}{4k} - q_0\right) + \frac{Np_0(k-1)^2}{k}.$$

To see the impact of  $k$  for a fixed  $p_0$ , when  $k$  is large, ISP tends to decrease its incentive to EROs and provide SUs having smaller willingness to pay with low quality **edge** service. However, for small  $k$  (i.e., the quality of **edge** service becomes relatively higher), ISP leases more resources from EROs by giving higher incentive to maximize its revenue. In the proof, we show that the revenue  $\pi^*(q)$  is a unimodal function<sup>1</sup> of  $q$ , which clearly presents the tradeoff between the cost for leasing edge resource from EROs and the income from SUs subscribing to **edge** service. ISP decides its incentive for EROs so as to maximize the revenue and this results in the equilibrium incentive.

### 3.4 Proofs

*Proof of Proposition 3.1.* Proposition 3.1 shows the utility-maximizing strategy of an SU. We first consider the case where  $\alpha_c \geq \alpha_e$ . Let  $\gamma_e$  be the willingness to pay for an SU who takes the **core** service. If SU with  $\gamma_c$  chooses the **core** service, it means that the utility of SU is positive and it is higher than that with **edge** service, as follows.

$$\gamma_e\alpha_c - p_c \geq 0 \quad (13)$$

$$\gamma_c\alpha_c - p_c \geq \gamma_e\alpha_e - p_e. \quad (14)$$

By combining (13) and (14), and using the assumption  $\alpha_c \geq \alpha_e$ , the selection of the **core** service maximizes SU's utility if and only if its willingness to pay  $\gamma$  satisfies the following:

$$\gamma > \max\left(\frac{p_c - p_e}{\alpha_c - \alpha_e}, \frac{p_c}{\alpha_c}\right).$$

1. A function  $f(x)$  is a unimodal function if for some value  $m$ , it is monotonically increasing for  $x \leq m$  and monotonically decreasing for  $x \geq m$ .

Similarly, if SU chooses the **edge** service, it means that when it subscribes **edge** service, the utility of SU is positive and it is higher than that with **core** service, as follows.

$$\gamma_e\alpha_e - p_e \geq 0$$

$$\gamma_e\alpha_e - p_e \geq \gamma_e\alpha_c - p_c,$$

where  $\gamma_e$  is the willingness to pay for an SU who subscribes **edge** service. This results in the condition of  $\gamma$  as following:

$$\frac{p_e}{\alpha_e} < \gamma \leq \frac{p_c - p_e}{\alpha_c - \alpha_e}.$$

Finally, the remaining SUs would not choose any services. A similar analysis can be applied to the case when  $\alpha_c < \alpha_e$ . This completes the proof.  $\square$

*Proof of Proposition 3.2.* We first describe our proof strategy, followed by detailed proof. We claim that the equilibrium point is  $(p_c^*, p_e^*) = (\frac{1}{2}p_0 + \frac{1}{2}\alpha_c, \frac{1}{2}\alpha_e)$  and show that  $\pi(p_c^*, p_e^*) \geq \pi(p_c, p_e)$  for all  $(p_c, p_e) \neq (p_c^*, p_e^*)$ . One technical challenge is that ISP's revenue  $\pi$  has a different shape depending on the condition of  $(p_c, p_e)$  as shown in (6). Thus, we consider all conditions and show that  $(p_c^*, p_e^*)$  is the equilibrium point.

We introduce the function  $\bar{\pi}$  defined for all  $(p_c, p_e)$  regardless of the conditions.

$$\bar{\pi}(p_c, p_e) = \sum_i \bar{\pi}_i(p_c, p_e) \mathbb{1}\{(p_c, p_e) \in A_i\}, \quad (15)$$

where  $\bar{\pi}_i(p_c, p_e) := \frac{1}{N}\pi_i(p_c, p_e) + B$ .  $\frac{1}{N}$  and  $B$  are constants for simplicity. Since (15) has the same optimal solution as  $\pi(p_c, p_e)$ , we are interested in the solution of (15). Additionally, we define

$$(z_1(t), z_2(t)) := ((1-t)x + tp_c^*, (1-t)y + tp_e^*), \quad (16)$$

where  $x$  and  $y$  are arbitrary non-negative real numbers and  $t$  is a real number in  $[0, 1]$ .  $(z_1(0), z_2(0)) = (x, y)$  can represent all possible values of  $(p_e, p_c)$ . Thus, we will show that  $\bar{\pi}(z_1(t), z_2(t))$  is maximized when  $t = 0$ .

We now present a key lemma whose proof is in Appendix.

**Lemma 3.1.** *For any given  $x, y$ ,  $\bar{\pi}(z_1(t), z_2(t))$  is a non-decreasing function of  $t \in [0, 1]$ .*

From Lemma 3.1, we can derive the following. For all  $(x, y) \in [0, \infty) \times [0, \infty)$ ,

$$\begin{aligned} \pi(x, y) &= N\bar{\pi}(z_1(0), z_2(0)) - B \\ &\leq N\bar{\pi}(z_1(1), z_2(1)) - B = \pi\left(\frac{1}{2}p_0 + \frac{1}{2}\alpha_c, \frac{1}{2}\alpha_e\right). \end{aligned}$$

Since  $\bar{\pi}(z_1(t), z_2(t))$  is non-decreasing of  $t \in [0, 1]$ , it is maximized when  $t = 1$  and this results in that the optimal solution of  $\bar{\pi}$  is  $(p_c^*, p_e^*) = (\frac{1}{2}p_0 + \frac{1}{2}\alpha_c, \frac{1}{2}\alpha_e)$ . This completes the proof.  $\square$

*Proof of Corollary 3.1.* We get the ISP's decision of prices in Stage II from Proposition 3.2. By substituting  $(p_c^*, p_e^*)$  into the conditions of regions  $A_3$  and  $A_4$  in (5), we can derive the contradiction, as follows. We first consider the case where  $\alpha_c \geq \alpha_e$ . Suppose that  $(p_c^*, p_e^*) \in A_3$ , then  $p_c^* < \frac{\alpha_c p_e^*}{\alpha_e}$  should hold. However, the right hand side is  $\frac{\alpha_c}{2} > p_c^*$ , which is the contradiction. Suppose that  $(p_c^*, p_e^*) \in A_4$ . In this case  $p_e^* = \frac{\alpha_e}{2} < \alpha$  which is the contradiction. A similar analysis can be applied to the case when  $\alpha_c < \alpha_e$ , and this completes the proof.  $\square$



*Proof of Theorem 3.1.* By backward induction, we have shown the equilibrium analysis of Stage II and that of Stage I-B, sequentially. Now, we provide the equilibrium of the ISP at the Stage I-A. We first show that  $\pi^*$  is a differentiable and strictly concave function so we can narrow the candidates of  $q^*$  as the local maximum points of  $\pi_1^*(q)$  and  $\pi_2^*(q)$  individually. Finally, we check the feasibility of all local maximum points and derive the equilibrium incentive by comparing those candidate points.

Depending on the condition C0 of  $q$ ,  $\pi^*$  has a different shape as presented (10). Let  $\bar{q}$  denote the boundary of  $q$  between R1 and R2, then  $\bar{q} = \alpha_e^{-1}(\alpha_c - p_0)$ . In each regime, substituting (7) into (6), we obtain  $\pi_1^*(q)$  and  $\pi_2^*(q)$  as follows:

$$\begin{aligned}\pi_1^*(q) &= \frac{Np_0^2}{4(\alpha_c - \alpha_e(q))} - \frac{N(2p_0 - \alpha_c)}{4} - \frac{bNq^2}{q_0}, \\ \pi_2^*(q) &= \frac{N\alpha_e(q)}{4} - \frac{bNq^2}{q_0}.\end{aligned}\quad (17)$$

We first show that  $\pi^*(q)$  is continuous for  $q \in [0, q_0]$ . Since  $\alpha_e$  is continuous, it is trivial that  $\pi_2^*$ . For  $q < \bar{q}$ ,  $\alpha_c - \alpha_e(q) > 0$  holds, and thus  $\pi_1^*$  is also continuous. For  $\bar{q}$ ,  $\pi_1^*(\bar{q}) = \pi_2^*(\bar{q})$ , therefore  $\pi^*$  is continuous for  $q \in [0, q_0]$ . Similarly, since  $\alpha_e$  is differentiable for  $q \in [0, q_0]$ , both  $\pi_1^*$  and  $\pi_2^*$  are differentiable. At the boundary of two regimes,  $\pi_1^{*'}(\bar{q}) = \pi_2^{*'}(\bar{q})$  holds so that  $\pi^*$  is also differentiable.

From the fact that  $\alpha_e(q)$  satisfies  $\alpha_e'(q) > 0$  and  $\alpha_e''(q) < 0$ , we can easily induce that  $\pi_2^*(q)$  is strictly increasing and strictly concave. Furthermore,  $\pi_1^*(q)$  is also strictly increasing and strictly concave when  $q < \bar{q}$ . To summarize these facts, we can see that  $\pi^*$  is strictly increasing and strictly concave with the unique local optimal point  $q_\pi^*$  which satisfies  $\frac{d\pi^*(q_\pi^*)}{dq} = 0$ . It is obvious that  $q_\pi^*$  would be a local optimal point of  $\pi_1^*$  or  $\pi_2^*$  from the definition of  $\pi^*$ . Since the both  $\pi_1^*$  and  $\pi_2^*$  are strictly increasing and strictly concave, each  $\pi_i$  has its unique local optimal point denoted as  $q_1^*$  and  $q_2^*$ , respectively. We can find the optimal  $q_1^*$  and  $q_2^*$  by differentiating  $\pi_1^*(q)$  and  $\pi_2^*(q)$  with relation to  $q$ , respectively. From (17) the derivatives of  $\pi_i^*(q)$  are:

$$\begin{aligned}\frac{d\pi_1^*}{dq}(q) &= \frac{Np_0^2\alpha_e'(q)}{4(\alpha_c - \alpha_e(q))^2} - \frac{2bNq}{q_0}, \\ \frac{d\pi_2^*}{dq}(q) &= \frac{N\alpha_e'(q)}{4} - \frac{2bNq}{q_0}.\end{aligned}\quad (18)$$

Thus,  $q_1^*$  is the unique solution of following equation:

$$8bq(\alpha_c - \alpha_e(q))^2 = p_0^2q_0\alpha_e'(q),$$

and  $q_2^*$  is the unique solution of following equation:

$$8bq = q_0\alpha_e'(q).$$

Now, we want to get the optimal point  $q^*$  for  $\pi^*(q)$  when  $q \in [0, q_0]$ . We can easily show that  $q^* > 0$  since  $\pi_1^*(q)$ ,  $\pi_2^*(q)$ , and  $\pi^*(q)$  are strictly increasing when  $q = 0$  from (18). Therefore, we can conclude that there are only two following candidates for  $q^*$ :  $\min(q_0, q_1^*)$  and  $\min(q_0, q_2^*)$ . It is hard to get a clear closed form of  $q^*$  from the complexity of our model. However, since the definition of  $q^*$  is that the maximal point of  $\pi^*(q)$  in  $[0, q_0]$ , we can say that  $q^* = \arg \max_{q \in \{\min(q_1^*, q_0), \min(q_2^*, q_0)\}} \pi^*(q)$ .  $\square$

*Proof of Proposition 3.3.* We get the equilibrium of ISP's strategies in Theorem 3.1. From Theorem 3.1 we can get the equilibrium behaviors of SUs and EROs for given incentive and prices by ISP.

Thus, we can derive SUs' service subscription strategy at equilibrium by substituting  $p_c^*$ ,  $p_e^*$ ,  $q^*$  and  $\alpha_e(q^*)$  into Proposition 3.1, and EROs' strategy from (8).

The strategies of EROs and SUs depend on condition C0. When C0 holds, as presented in the proof of Theorem 3.1,  $\alpha_c \geq \alpha_e$  holds, SUs strategy results in (i) in Proposition 3.1. Otherwise, there are two cases whether  $\alpha_c \geq \alpha_e$  holds or not. When it holds,  $\frac{p_e - p_c}{\alpha_c - \alpha_e} \geq 1$  and thus no SUs use **core** service, and SUs with higher  $\gamma$  than  $\frac{1}{2}$  use **edge** service. When  $\alpha_c \geq \alpha_e$  does not hold,  $\frac{p_c}{\alpha_c} > \frac{p_e - p_c}{\alpha_e(q^*) - \alpha_c}$  and thus there is no SU who subscribes to **core** service, and SUs with higher  $\gamma$  than  $\frac{1}{2}$  subscribe to **edge** service. We can get EROs' equilibrium strategy by substituting  $q^*$  into (8).  $\square$

*Proof of Proposition 3.4.* The proof of the proposition follows two steps. First, we will show that (2) satisfies the condition for  $f$  in (1). Second, by using the result of Theorem 3.1, we present the closed form expression of equilibrium behavior for given the assumption of service quality.

From (2), the derivative of  $\alpha_e(m)$  is:

$$\alpha_e'(m) = \frac{\beta\alpha_e aN}{(am + N)^2}.$$

Since it is positive so  $\alpha_e$  is strictly increasing function of  $m$ . We can easily get the result that  $\alpha_e$  is strictly concave function by showing twice differentiated function of  $\alpha_e$  is negative. Thus, the result for Theorem 3.1 can be applied to the case with the quality of service (2).

Let  $q_1^*$  and  $q_2^*$  denote the local maximum of  $\pi_1^*(q)$  and  $\pi_2^*(q)$ , respectively. We can find the optimal  $q_1^*$  and  $q_2^*$  by differentiating  $\pi_1^*(q)$  and  $\pi_2^*(q)$  with relation to  $q$ , respectively. From Propositions 3.1 and 3.2, the explicit expressions of the derivatives of  $\pi_i^*(q)$  are:

$$\begin{aligned}\frac{d\pi_1^*}{dq}(q) &= \frac{ab\beta\alpha_e Nq_0p_0^2}{4(\alpha_c(1-\beta)abq + \alpha_cq_0)^2} - \frac{2bNq}{q_0}, \\ \frac{d\pi_2^*}{dq}(q) &= \frac{ab\beta\alpha_e Nq_0}{4(abq + q_0)^2} - \frac{2bNq}{q_0}.\end{aligned}$$

Thus, (11) and (12) in Theorem 3.1 can be replaced by following two equations, respectively.

$$\begin{aligned}8bq(\alpha_c - \alpha_e(q))^2 &= p_0^2q_0\alpha_e'(q), \\ 8bq &= q_0\alpha_e'(q).\end{aligned}$$

This completes the proof.  $\square$

## 4 NUMERICAL RESULTS

In this section, we provide a set of numerical results to draw practical implications based on our analytical results. We show the impacts of **edge** service on the equilibrium of fog computing market, where the equilibrium is affected by the interactions between the numerically quantifying economic metrics such as edge/core costs, efficiency, and maximum QoS. To make our numerical result more analytically interpretable, we use the model of service quality with (2). For simplicity, we assume that the number of SUs is equivalent to that of EROs so that  $N = 100$ . Moreover, we choose  $a = 5$  and  $\beta = 1$  to model a network where there are 50 number of sharing EROs and the edge service QoS is

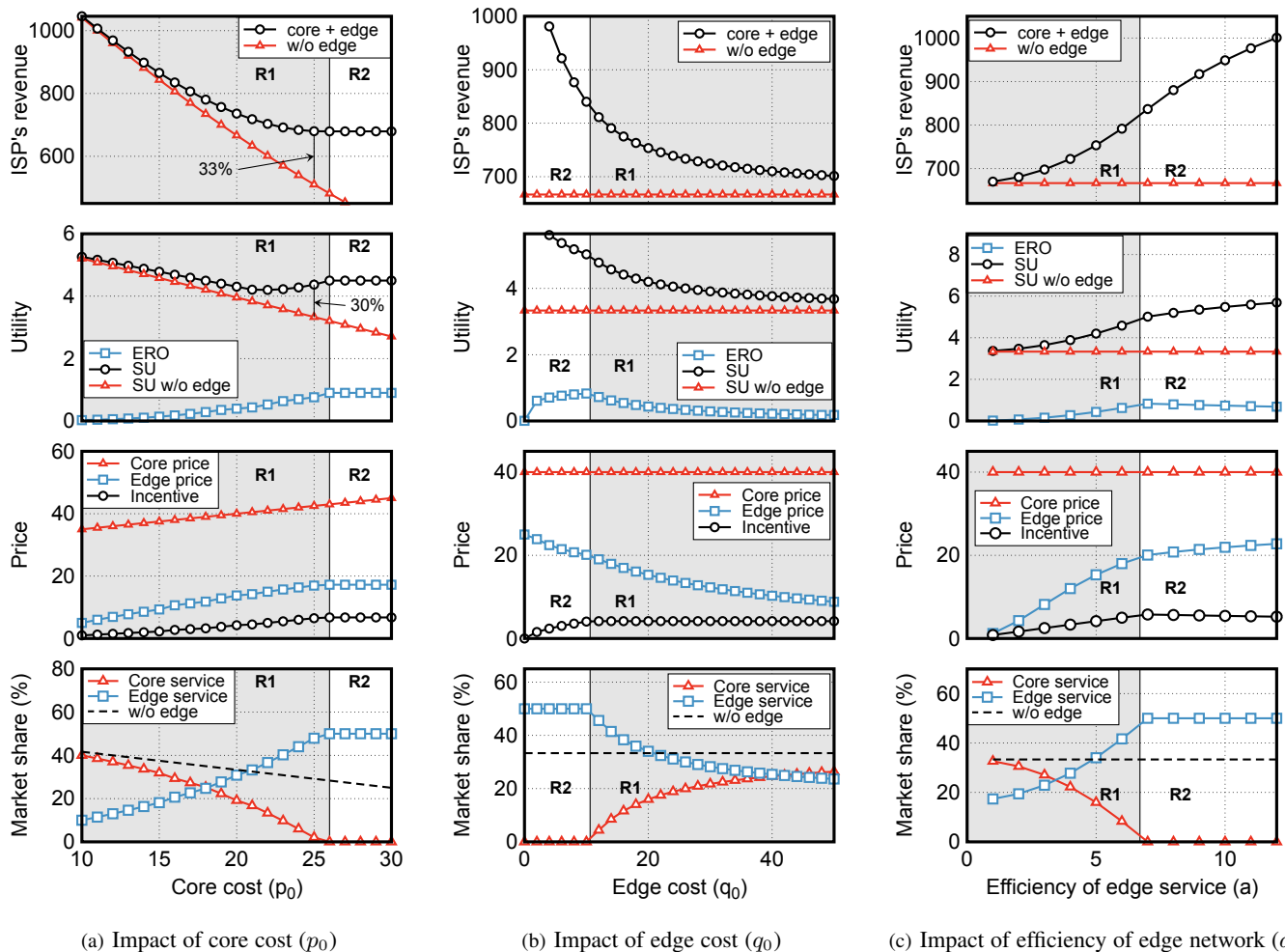


Fig. 7. ISP's revenue, SU's total utility, and market share of core and edge services for varying core cost ( $p_0$ ) and edge cost ( $q_0$ ) where  $\beta = 1$ .

70% of the core service QoS. From AT&T's monthly data plan of IoT cellular service [39] and incentive policy of Karma [31], we choose  $p_0 = \$25$  and  $q_0 = \$25$  where an ERO shares its resource to 25 SUs. We assume that the QoS of core service  $\alpha_c$  is 60 for which SU with  $\gamma = 1$  is willing to pay \$60.

#### 4.1 Impact of edge service on ISP's Revenue, SUs' and EROs' Utilities, and Market Shares

In order to show the impact of adding edge service, we compare the case where ISP provides both edge and core services from when it provides just core service. We first study the case where the maximum QoS of edge service equals to that of core service (i.e.,  $\beta = 1$ ). This case corresponds to the applications in which SUs have no preference for edge or core services if edge resources are fully shared (e.g., network connectivity application). We later show the results for other varying values of  $\beta$  in Section 4.2.

**Observation 1.** edge service improves ISP's revenue by 33%. The plots in the first row of Fig. 7 show the revenue of ISP. In fog-computing market, ISP provides SUs with core and edge services simultaneously, but for comparison, we also draw the ISP's revenue when ISP provides only core service. In all plots in the first row, ISP's revenue is overwhelming in the cases where the ISP provides only core service, and this verifies the result of

Theorem 3.1. In our environmental parameter setting, the revenue of ISP increases by 33% compared with the case where ISP provides only core service.

**Observation 2.** SUs' utility is improved by 30% on average, and EROs' utility also increases. Additionally, although ISP tries to maximize its own revenue by setting appropriate prices and incentive, SUs' average utility increases, as shown in plots in the second row of Fig. 7. In our setting, an SU's total utility increases by 30% compared with the case where ISP only provides core service. In addition to this, the average utilities of EROs also increase by incentive given by ISP, so that we conclude that edge service is always beneficial to all players in the fog computing market.

**Observation 3.** edge service reduces the ISP's usage of core resource by 93%. The plots in the bottom of Fig. 7 include two-fold information: (i) the market share between core and edge services in fog-computing market, and (ii) that of core service when there is no edge service (dotted line in the plots). The number of SUs subscribing to core service in (i) is always smaller than that in (ii). ISP uses only 7% of the core resource compared to (ii). It is because SUs, whose willingness to pay is relatively low, prefer subscribing to edge service with lower price despite its lower QoS. Thus, providing edge service can reduce the cases requiring core resources.

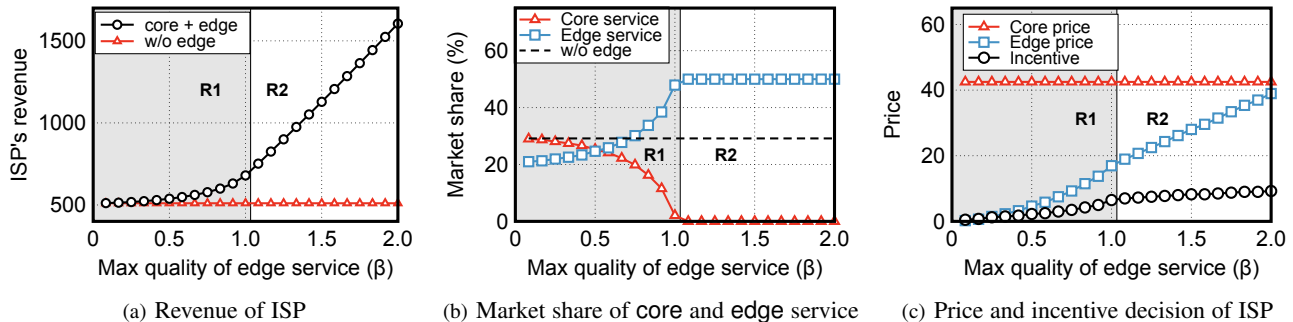


Fig. 8. Impact of maximum quality of edge service ( $\beta$ ) on ISP's revenue, market share, and price decision.

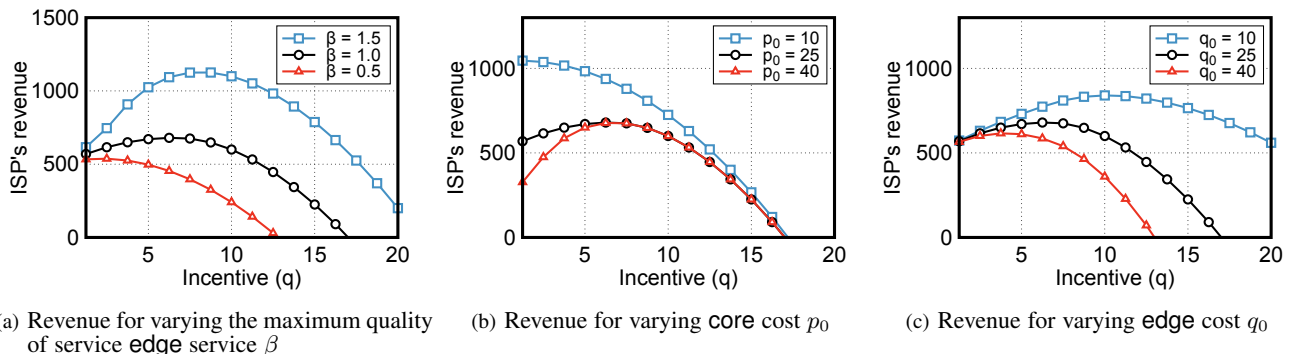


Fig. 9. Impact of ISP's decision regarding incentive on the revenue.

## 4.2 Impact of Other Environmental Parameters

In this section, we study the impact of other environmental parameters such as core and edge costs, efficiency of edge service, and the maximum quality of edge service on the revenue of ISP, SU's utility, and the strategy selection of each player at the equilibrium. We also investigate the impact of ISP's decision on the incentive for EROs on the revenue of ISP.

**Observation 4.** *How ISP's equilibrium strategy depends on the environmental parameters?* In the fog-computing market, ISP's strategy decision is classified into two regimes, depending on given environmental parameters. In Fig. 7, shaded area refers to regime R1 and the unshaded area refers to regime R2. As edge resource becomes more efficient (i.e.,  $a$ ,  $p_0$  increase or  $q_0$  decreases), ISP prefers providing the edge service, and it finally becomes in the regime R2 and provides only edge service. For example, in our parameter setting in Fig. 7(a), the threshold value of the core cost ( $p_0$ ) is \$24, and if the core cost exceeds the threshold, all SUs subscribe to the edge service.

**Observation 5.** *Efficient edge resource improves the revenue of ISP.* Figs. 7(b) and 7(c) show the results with varying  $q_0$  and  $a$ , respectively. When ISP leases edge resources by giving a small amount of incentive (i.e., small  $q_0$ ), ISP provides the edge service more aggressively to earn higher revenue. In our parameter setting, as  $q_0$  decreases from \$25 to \$10, the revenue increases by 15%. When  $a$  is high, the QoS of edge service increases aggressively relative to the number of sharing-EROs, which results in the increase of SUs subscribing to edge service as shown at the bottom plot of Fig. 7(c). As  $a$  grows from 5 to 10, the revenue increases by 25%.

**Observation 6.** *High QoS in edge services (i.e., when  $\beta$  is large) leads to the increase of ISP's revenue.* Fig. 8 shows the impact of maximum quality of edge service  $\beta\alpha_c$ . As  $\beta$  grows, ISP

leases more resources from EROs by giving higher incentive, and provides SUs with high quality edge service to maximize the revenue. In our parameter setting, when  $\beta = 1.75$ , ISP decides the incentive as \$8.75 (see Fig. 8(c)) where 35 out of 100 EROs share their resources. This results in the QoS of edge service overwhelming that of core service, and no SU subscribes to core service, i.e., ISP's strategy is in regime R2 (see Fig. 8(b)). As shown in Fig 8(a), compared to the case when  $\beta = 1$ , ISP's revenue increases twice, and we conclude that high QoS in edge service leads to the increase of ISP's revenue.

**Observation 7.** *ISP's decision of incentive has a tradeoff between income from SUs and cost for incentivizing EROs.* As Fig. 9(a) shows, we see how the incentive affects the revenue of ISP. This tradeoff is between the cost for leasing edge resource from EROs and the income from SUs subscribing to edge service. ISP selects incentive which maximizes the revenue of ISP and this results in the equilibrium incentive. In Fig. 9(a), as  $\beta$  increases, the incentive at equilibrium also increases. Similarly, as core (resp. edge) cost increases from \$10 to \$40, the ISP increases (resp. decreases) the incentive, and the revenue is reduced by 35% (resp. 27%), as shown in Figs 9(b) and 9(c).

## 5 CONCLUSION

It is expected that a huge number of edge devices will be deployed by individuals in the near future. In this paper, we model/analyze an emerging edge resource market, which we call fog computing market, where ISP which provides a platform of fog computing, behaves as a mediator to lease edge resources from EROs and provide services to SUs. By modeling this market as an ISP-platformed two-stage embedded dynamic game, we prove the existence of a fog computing feasibility region, where fog computing market increases ISP's revenue as well as utilities of EROs and SUs.

## APPENDIX

*Proof of Lemma 3.1.* In this proof, we will show that  $\bar{\pi}(z_1(t), z_2(t))$  is the non-decreasing function of  $t \in [0, 1]$ . Since  $\bar{\pi}$  has a different shape depending on the condition of  $(z_1(t), z_2(t))$  we will show the continuity of  $\bar{\pi}$  and show the non-decrease of  $\bar{\pi}_i(z_1(t), z_2(t))$  by using the continuity of this function. From equation (15), we can get  $\bar{\pi}(z_1(t), z_2(t))$  as follows.

$$\bar{\pi}(z_1(t), z_2(t)) = \sum_{i=1}^4 \bar{\pi}_i(z_1(t), z_2(t)) \cdot \mathbb{1}\{(z_1(t), z_2(t)) \in A_i\}$$

From (6),  $\pi$  is continuous because all  $\pi_i$ s are continuous functions. We need to check the boundary between the partitions. We first consider the boundary between  $A_1$  and  $A_2$  when  $\alpha_c > \alpha_e$ . From (5), at the boundary  $p_c = p_e + \alpha_c - \alpha_e$  holds. By substituting  $p_c$  into  $\pi_1$  and  $\pi_2$ , we can easily show that  $\pi_1 = \pi_2$  at the boundary. In the same way, we can show that  $\pi_i$ s are continuous at all boundaries of partition  $A_i$  for all  $i$ . The continuity of  $\bar{\pi}(z_1(t), z_2(t))$  follows the continuity of  $\pi(p_c, p_e)$  and the fact that  $z_1(t)$  and  $z_2(t)$  are also continuous over  $t$ .

Now, we show the non-decrease of  $\bar{\pi}(z_1(t), z_2(t))$  by the using continuity of the function. In order to show that  $\bar{\pi}(z_1(t), z_2(t))$  is a non-decreasing function of  $t$ , it is enough to show that  $\bar{\pi}_i(z_1(t), z_2(t))$  is a non-decreasing function of  $t$  for all  $i$ , because  $\bar{\pi}$  is continuous.

We rewrite  $\bar{\pi}_i(p_c, p_e)$  as the summation of quadratic expressions for given environmental parameters  $\alpha_c$ ,  $\alpha_e$ , and,  $p_0$ . For example, when  $\alpha_c > \alpha_e$ ,  $\pi_1(p_c, p_e)$  is written as follows.

$$\begin{aligned} \bar{\pi}_1(p_c, p_e) &= (p_c - p_0) \left(1 - \frac{p_c - p_e}{\alpha_c - \alpha_e}\right) + p_e \left(\frac{p_c - p_e}{\alpha_c - \alpha_e} - \frac{p_e}{\alpha_e}\right) \\ &= -\frac{\alpha_c}{(\alpha_c - \alpha_e)\alpha_e} \left(p_e - \frac{\alpha_e p_c - \frac{1}{2}\alpha_e p_0}{\alpha_c}\right)^2 \\ &\quad - \frac{1}{\alpha_c} \left(p_c - \frac{1}{2}(p_0 + \alpha_c)\right)^2 \\ &\quad + \frac{1}{4} \cdot \frac{1}{\alpha_c - \alpha_e} p_0^2 - \frac{1}{2} p_0 + \frac{1}{4} \alpha_c. \end{aligned} \quad (19)$$

From above, we can easily show that the prices  $p_{c1}^*$  and  $p_{e1}^*$  which maximize  $\bar{\pi}_1(p_c, p_e)$  satisfy following.

$$p_{c1}^* = \frac{1}{2}(p_0 + \alpha_c), \quad p_{e1}^* = \frac{\alpha_e p_{c1}^* - \frac{1}{2}\alpha_e p_0}{\alpha_c} = \frac{1}{2}\alpha_e. \quad (20)$$

By substituting (20) to (19), we can get the maximum revenue of ISP for partition  $A_1$ , as follows.

$$\bar{\pi}_1(p_{c1}^*, p_{e1}^*) = \frac{1}{4(\alpha_c - \alpha_e)} p_0^2 - \frac{1}{2} p_0 + \frac{1}{4} \alpha_c.$$

To show the non-decrease of  $\bar{\pi}_1((z_1(t), z_2(t)))$ , we substitute (16) to (19) and we get  $\bar{\pi}_1((z_1(t), z_2(t)))$ , as follows.

$$\begin{aligned} \bar{\pi}_1(z_1(t), z_2(t)) &= \frac{-(1-t)^2 \alpha_c}{(\alpha_c - \alpha_e)\alpha_e} \left(p_e - \frac{\alpha_e p_c - \frac{1}{2}\alpha_e p_0}{\alpha_c}\right)^2 \\ &\quad - (1-t)^2 \frac{1}{\alpha_c} \left(p_c - \frac{1}{2}(p_0 + \alpha_c)\right)^2 \\ &\quad + \frac{1}{4} \cdot \frac{1}{\alpha_c - \alpha_e} p_0^2 - \frac{1}{2} p_0 + \frac{1}{4} \alpha_c. \end{aligned}$$

It is obvious  $\bar{\pi}_1((z_1(t), z_2(t)))$  is a non-decreasing function of  $t \in [0, 1]$ . Similarly, we can show the non-decrease of

$\bar{\pi}_2((z_1(t), z_2(t)))$ ,  $\bar{\pi}_3((z_1(t), z_2(t)))$ , and  $\bar{\pi}_1((z_1(t), z_2(t)))$  where  $\alpha_c \leq \alpha_e$ . For  $\bar{\pi}_4((z_1(t), z_2(t)))$ , it is trivial because it is constant. All  $\bar{\pi}_i((z_1(t), z_2(t)))$  are the non-decreasing function of  $t$  and  $\bar{\pi}$  is continuous, and thus  $\bar{\pi}$  is the non-decreasing function of  $t$ . This completes the proof.  $\square$

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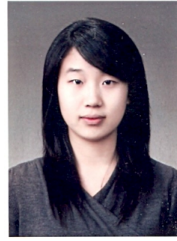
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