On the Stability of ISPs' Coalition Structure: Shapley Value based Revenue Sharing

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***Abstract*—The Internet is a complex system, consisting of different economic players in terms of access/transit connection and content distribution, which are typically selfish and try to maximize their own profits. Due to this different perspective of economic interest as well as dynamic changes of the Internet market, a certain degree of techno-economic inefficiency has naturally been observed, e.g., unstable peering and revenue imbalance among content, eyeball, and transit ISPs (Internet Service Providers). At the center of this issue is "good" revenue sharing among them. Recently, revenue sharing based on the notion of Shapley Value (SV) from cooperative game theory has been applied to address the afore-mentioned issue, shedding light upon many nice properties which have been used not only to understand the current Internet eco-system but also to predict its future. However, the positive features from the SV based revenue sharing can be practically feasible only when the providers agree to form a grand coalition, which may not hold in practice. In this paper, we first investigate the conditions under which the grand coalition is stable under SV by classifying the network into two cases: under-demanded and over-demanded. We then study the gap between the conditions of the grand coalition's stability and optimal coalition structures (i.e., coalition structures that maximize the aggregate revenue of ISPs).**

I. Introduction

The Internet is a system where the entities such as EUs (End Users) and content/eyeball/transit ISPs (Internet Service Providers)[1], having different economic perspectives, compete and cooperate in a highly complex manner. Eyeball/transit ISPs connect EUs to the Internet, and content ISPs inject and deliver content data into the Internet [1], e.g., videos, web pages, and files. The major interest of the providers, which is often selfish, is to maximize their profits, sometimes incurring techno-econo inefficiency in the Internet. For example, ISPs' selective peering with other ISPs may have negative impact on Internet's connectivity. Different ISPs express economic complaints on revenue imbalance among them, which becomes a major obstacle to evolvability of the Internet. One of the central issues regarding such complaints is how to distribute the revenue from the users among the providers in a fair manner.

Motivated by the above, there have been recent research efforts on fair and efficient revenue sharing among providers, using the notion of Shapley value (SV) [2] from cooperative game theory. The SV based revenue sharing enforces the profit distribution at a multilateral, global level, rather than a bilateral, local level, leading to nice features in terms of fairness, efficiency, and interconnection incentives, see e.g., [3] and [4]. The major messages include *(i)* multi-lateral settlements among ISPs are more preferable than bi-lateral ones and *(ii)* even selfish behaviors of the ISPs may yield globally optimal routing and interconnection decisions. In addition to SV's application to providers' settlements, it has also been applied to many other network-economic problems, e.g., peer-assisted services [5], viral marketing [6], and virtual infrastructure sharing [7].

The SV is a fair payoff distribution in cooperative game theory based on the assumption that a *grand coalition* (i.e., coalition containing all players) is agreed by the players. However, the question of whether the providers would form the grand coalition is largely open. In [3], the grand coalition seems to be formed under a simple model which is adopted mainly for tractable analysis. When more realistic features are added to the model, the grand coalition may not be agreed upon by the providers, which in turn disproves the efficacy of the SV based revenue sharing. This motivates us to perform in-depth studies of grand coalition's stability.

Of many important realistic features required to revive in the modeling and analysis, we focus on the case when user demands exceed network capacity. This *over-demanded* network seems practically important due to the recent trends in the Internet access, including wireless and wired parts, where edge devices, e.g., smart TV and smart phones, are becoming more traffic-aggressive and exponentially growing number of contents are injected into the Internet by CPs and consumed by EUs. As will be presented in this paper, SV's features in over-demanded networks are in stark contrast to those in under-demand ones (which is the major premise of the earlier work), especially as for the stability of grand coalition under the SV based revenue sharing. In the over-demanded networks, the users' QoS is naturally degraded (see, e.g., [8] for the correlation between the server delays and the number of users), depending on so-called *over-demand policy*, and thus the "actual" yields (a part of the demand that are provided)

[1]ISP is sometimes called just 'provider' throughout this paper.

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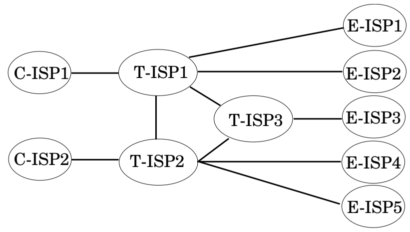

Fig. 1. Network Model: Example

vary with the adopted over-demand policy, presumably leading to the complicated economic interplay among the providers.

The impacts of user demands and over-demand policies are not restricted to the stability of grand coalition under SV, but are also closely connected to the *social welfare* of the grand coalition (i.e., the aggregate value among the providers). While the stability is related to coalitions' sustainability which is a social challenge posed by individual and selfish aspects of providers, the social welfare reflects coalitions' ultimate value that is sustained by the cooperative aspects of providers. Thus, mandating an over-demand policy which maximizes social welfare could be a communal objective in the Internet.

Our research aims to address the following questions:

1) *Stability.* In each of under-demanded and over-demanded networks, under what conditions is the grand coalition stable with the SV based revenue sharing and what are their engineering implications?
2) *Optimal coalition structure.* Which coalition structures are the ones that maximize the social welfare and, particularly, under what conditions (of network and demand configurations) is the grand coalition such an optimal coalition structure?
3) *Impact of over-demand policy.* What are the coupling relations between conditions for stability and optimal coalition structure and over-demand policies adopted by the eyeball ISPs?

In this invited paper, we present our partial answers to the above questions by presenting our preliminary results. We end this section by summarizing the related work additionally to those mentioned earlier. The research on the revenue sharing mechanisms based on proportional fairness and NBS (Nash Bargaining Solution) has been made in [9]–[11]. In the view of the stability of the coalition, the authors in [5] show that the grand coalition could be unstable in peer-assisted services with multi-providers even if stability is guaranteed under a single provider. The authors in [12], [13] study optimal coalition structures, and its applications to skill game [14] and vehicle routing [15] have been presented.

II. Model

A. Network Model and Notations

Network and Transit ISPs. We consider a network consisting of a set $\mathcal{T}$ of transit ISPs, a set $\mathcal{C}$ of content ISPs, and a set $\mathcal{B}$ of eyeball (or access) ISPs, where we denote by $\mathcal{N} = \mathcal{C} \cup \mathcal{T} \cup \mathcal{B}$ the set of all "providers". Transit ISPs offer connectivity between eyeball ISPs and content ISPs. For simplicity we assume that a content ISP (and also an eyeball ISP) is connected to just one transit ISP and no direct connection between any content ISP and eyeball ISP exists. Note that a transit ISP may be connected to many eyeball ISPs and/or content ISPs. Transit ISPs are assumed to be fully connected, see Fig. 1 as an example.

Regions and Eyeball ISPs. Eyeball ISPs connect residential users to a transit ISP. Denote $\mathcal{R}$ as the set of all regions served by the set of eyeball ISPs $\mathcal{B}$. We also denote by B_r the eyeball ISP which covers the region $r \in \mathcal{R}$, where we assume that there does not exist a region covered by multiple eyeball ISPs. Let n_r be the link capacity between B_r and its connected transit ISP. The set of transit ISPs delivering traffic to the region r is denoted by $\mathcal{T}_r$. The content could be delivered from a content ISP to the region r via several transit ISPs, $\mathcal{T}_r$. We assume that the content takes the shortest path from its source content ISP to the requesting destination eyeball ISP. Then, in our model, since the transit ISPs form a full mesh topology, any content is delivered via at most two transit ISPs, i.e., $|\mathcal{T}_r| \leq 2, \forall r \in \mathcal{R}$.

Contents and Content ISPs. Let $\mathcal{Q}$ be the set of all contents in the network. Note that a content can be served by multiple content ISPs. Each region may have a different set of contents to download, for which we let $X_{r,q}$ be the user population in region r that has demand for content $q \in \mathcal{Q}$. Note that we assume that for a content q, users do not have preference for a content ISP serving q. Let $\gamma_{r,q} = X_{r,q}/X_r$ be the portion of region r's population that wants to access the content q. We denote $\mathcal{C}_q$ as the set of content ISPs that serve the content q and $\mathcal{T}_q$ as the set of transit ISPs that deliver the content q. Let $\mathcal{Q}_r \subset \mathcal{Q}$ be the set of contents demanded by the users in region r, and $\mathcal{C}_r$ be the set of content ISPs that serve at least one content in $\mathcal{Q}_r$. In other words, the set $\mathcal{C}_r$ is the union of the sets of content ISPs that serve the contents in $\mathcal{Q}_r$, thus $\mathcal{C}_r = \cup_{q \in \mathcal{Q}_r} \mathcal{C}_q$. Likewise, $\mathcal{T}_r = \cup_{q \in \mathcal{Q}_r} \mathcal{T}_q$. We let s_q be the average traffic volume of the content q.

Notation. We use the lower-case characters i, r, and q to index a content ISP, a region, and a content, respectively. To avoid notational complexity, in all notations we use subscripts for r and q and superscripts for i. Thus, we sometimes use C^i and $\mathcal{Q}^i$ to refer to the i-th content ISP and the set of contents served by C^i. For any coalition $S \subset \mathcal{N}$, we denote by $R[S]$ the set of regions restricted by the eyeball ISPs in S. Finally, to abuse notation, we use $i \in \mathcal{C}$ to mean $C^i \in \mathcal{C}$.

B. Charging, Revenue, and User Demand

Charging. Two main sources of revenue are the network access charges that are paid by the end users and garnered by the eyeball ISPs and the content access fee collected by the content ISPs that own the accessed contents. Denote by α_r the monthly access fee of the region r (and thus the eyeball ISP B_r), and by β_q the average per-content revenue earned by those content ISPs who are serving q.

User demand and over-demand policy. Consider the total potential traffic volume in region r, y_r, (i.e., "original" demand):

$$y_r = \sum_{q\in\mathcal{Q}_r} s_q X_{r,q} = \sum_{q\in\mathcal{Q}_r} s_q \gamma_{r,q} X_r.$$

Let $y_{r,q} = s_q X_{r,q}$ which corresponds to the traffic volume from the region r to access the content q. Eyeball ISPs will take some measures when the network is over-demanded, largely depending on the adopted over-demand policy, which technically consists of traffic engineering as well as QoS provisioning. For example, eyeball ISPs can give preference to the users that have longer subscription contracts or to the users of a special institution. In this paper, we assume that no such special cares are taken, and all the access traffic is treated neutrally, implying the *traffic-proportional* over-demand policy [2]; it reduces the traffic demand in proportion to the traffic volume generated by each content q. Then, we will have the *actual* user population $\bar{X}_{r,q}$ meaning the reduced user population after over-demand policy is applied, is given by:

$$\bar{X}_{r,q} = \min\left(X_{r,q}\frac{n_r}{y_r}, X_{r,q}\right).$$

Revenue. Let $Rev(B_r)$ and $Rev(C^i)$ be the revenues of the eyeball ISP B_r and content ISP C^i, respectively. Then the total revenue is $\sum_{r\in\mathcal{R}} Rev(B_r) + \sum_{i\in\mathcal{C}} Rev(C^i)$, where $Rev(B_r) = \alpha_r X_r$. As opposed to the eyeball ISPs' revenue, the content ISPs' revenue $Rev(C^i)$ is rather complex due to its dependence on demands, because in the over-demand case, just a part of users' content demand can be satisfied. Recall that $\bar{X}_{r,q}$ is the "actual" user population in region r demanding the content q and let $\bar{\gamma}_{r,q} = \bar{X}_{r,q}/X_r$. Then, the revenue of a content ISP $Rev(C^i)$ is given by:

$$\sum_{i\in\mathcal{C}} Rev(C^i) = \sum_{r\in\mathcal{R}}\sum_{q\in\mathcal{Q}_r} \beta_q \bar{X}_{r,q} = \sum_{r\in\mathcal{R}}\sum_{q\in\mathcal{Q}_r} \beta_q \bar{\gamma}_{r,q} X_r.$$

III. Preliminaries: Coalition Game, Shapley Value, and Stability

A. Coalition Game

In coalition games, there is a notion of *coalition* comprised of a set of players, in which the players act as a single group. The coalition endogenously forms a *coalition structure* (i.e., a partition) with respect to a coalition structure value [16]. We denote a coalition game with a coalition structure, by $(\mathcal{N}, v, \mathcal{P})$, where N is a set of players and the game has a transferable utility characterized by a worth function v, which is $v : 2^{\mathcal{N}} \to \mathbb{R}$ and $v(\emptyset) = 0$. The worth function associates with all coalitions $S \subseteq \mathcal{N}$, intuitively meaning for a given coalition the value generated by cooperation among the players in the coalition. A coalition structure $\mathcal{P}$ is a finite partition $\mathcal{P} = \{P_1, P_2, \cdots, P_m\}$ of $\mathcal{N}$. A coalition containing player i is denoted by $\mathcal{P}(i)$. In particular, the coalition structure $\mathcal{P} = \{\mathcal{N}\}$ is called *grand coalition*. A coalition game $(\mathcal{N}, v, \mathcal{P})$ is simply denoted by (N, v) if the coalition structure is the grand coalition. A coalition structure value is an operator φ which assigns values (or payoffs) to every player in game $(\mathcal{N}, v, \mathcal{P})$, then a coalition structure value (or simply value) for a player i is denoted by $\varphi^i(\mathcal{N}, v, \mathcal{P})$. We describe the concept of super-additivity stating that larger coalition achieves larger total worth.

Definition 1 (Super-additivity) For a game $(\mathcal{N}, v, \mathcal{P})$, the worth function v is super-additive if $v(S\cup T) \geq v(S) + v(T)$, for all $S, T \subset \mathcal{N}$ such that $S \cap T = \emptyset$.

B. Shapley Value

Shapley provides an axiomatic approach [2] to determine a coalition structure value φ, which reflects the following desirable properties:

Axiom 1 (Efficiency) $\sum_{i\in S}\varphi^i(\mathcal{N}, v, \mathcal{P}) = v(S),\ \forall S \in \mathcal{P}$.

Axiom 2 (Symmetry) If $j \in \mathcal{P}(i)$ and $v(S\cup\{i\}) = v(S\cup\{j\})$ for all $S \subseteq \mathcal{N}\backslash\{i, j\}$, then $\varphi^i(\mathcal{N}, v, \mathcal{P}) = \varphi^j(\mathcal{N}, v, \mathcal{P})$.

Axiom 3 (Additivity) For all worth functions v, v' and $i \in \mathcal{N}$, $\varphi^i(\mathcal{N}, v + v', \mathcal{P}) = \varphi^i(\mathcal{N}, v, \mathcal{P}) + \varphi^i(\mathcal{N}, v', \mathcal{P})$.

Axiom 4 (Dummy) If $v(S\cup\{i\}) = v(S)$ for all $S \subseteq \mathcal{N}$, then $\varphi^i(\mathcal{N}, v, \mathcal{P}) = 0$.

An intuitive explanation of the above axioms is as follows. (i) In efficiency, the coalition's worth is equal to the sum of the values of all players in the coalition, meaning there is no redundant worth on the profit distribution, (ii) in symmetry, the players have the same values if they make the same contribution to the coalition, (iii) in additivity, the value of each player is equal to the sum of the values from the separate worth functions, i.e., the value for a specific game does not affect the values for other games, and finally (iv) in dummy, the player who does not contribute to any coalition has no value.

It has been proved that the coalition structure value satisfying the above four axioms is *uniquely* determined for every coalitional game with the grand coalition $(\mathcal{N}, v)$, referred to as *Shapley value*, characterized as: for any player i,

$$\varphi^i(\mathcal{N}, v) = \frac{1}{|\mathcal{N}|!}\sum_{\pi\in\Pi}\Delta_i(v, S(\pi, i)), \tag{1}$$

where Π is the set of $|\mathcal{N}|!$ orderings of $\mathcal{N}$ and $S(\pi, i)$ is the set of players preceding i in the ordering π, and $\Delta_i(v, S)$ is the marginal contribution $\Delta_i(v, S)$ of player i for a coalition $S \subseteq \mathcal{N}\backslash\{i\}$, i.e., $\Delta_i(v, S) = v(S\cup\{i\}) - v(S)$. Thus, the Shapley value can be interpreted by the average marginal contribution over all orderings of players[3].

[2] We will later consider a different over-demand policy in Section V.

[3] Shapley value is only defined for the grand coalition. The axiomatic coalition structure value for any coalition structure $\mathcal{P}$ is called the Aumann-Drèze value (A-D value) [17]. Then, A-D value for a player $i \in S \in \mathcal{P}$ is also denoted by $\varphi^i(S)$ in this paper. There is no axiomatic difference between the Shapley value and the A-D value, but the only difference lies in the coalition structure in which the players are interested. Thus, for avoiding confusion, we will use the term of "Shapley value" for both of the axiomatic coalition structure values.

C. Stability

The stability of the coalition structure with respect to the coalition structure values has been studied in [16] (see, e.g., [18] for a tutorial), where [19] simplified the definition of stability in [16], presented in the following:

Definition 2 (Stability of Grand Coalition) The grand coalition is said to be *stable* for a game $(\mathcal{N}, v)$ with respect to the Shapley value φ, if and only if for all $S \subseteq \mathcal{N}$ there is a player $i \in S$ such that $\varphi^i(\mathcal{N}, v, \{\mathcal{N}\}) \geq \varphi^i(\mathcal{N}, v, \{S, \mathcal{N} \setminus S\})$.

A different description of Definition 2 is that there does not exist any coalition $\mathcal{P}$ that *blocks* $\{\mathcal{N}\}$, where we say that a coalition structure $\mathcal{P}$ blocks $\{\mathcal{N}\}$ (w.r.t. Shapley value φ), if and only if there exists a coalition $C \in \mathcal{P}$, such that $\varphi^i(\mathcal{N}, v, \{C, \mathcal{N} \setminus C\}) > \varphi^i(\mathcal{N}, v, \{\mathcal{N}\})$ for all players $i \in C$. To put it simply, Definition 2 implies that the grand coalition is not sustainable if there exists a more profitable coalition S for every player in S.

IV. Revenue Sharing Game (RSG) and Stability

A. RSG and Shapley Value

We now apply a coalition game summarized in Section III to the problem of sharing the revenues among the providers in our case, which we call *revenue sharing game (RSG)* throughout this paper. In RSG, we first build the worth function $v(\cdot)$ for an arbitrary coalition, using the revenue earned by the providers in the coalition, followed by characterization of the Shapley value.

The worth function of a coalition, say S, is the summation of revenues from the eyeball and the content ISPs, i.e.,

$$\begin{aligned} v(S) &= \sum_{r \in \mathcal{R}[S]} Rev(B_r) + \sum_{i \in \mathcal{C}[S]} Rev(C^i) \\ &= \sum_{r \in \mathcal{R}[S]} \left(\alpha_r X_r + \sum_{q \in \mathcal{Q}_r[S]} \beta_q \bar{X}_{r,q} \right). \end{aligned} \quad (2)$$

As discussed in [4], the worth for a coalition can be separately characterized by each revenue source for the network and content access fee by the eyeball and content ISPs, respectively. In such a case, we can appropriately decompose the network topology in S into the ones from the perspective of a region r and a tuple (region r, content q). Let $S_{r,q} \subset S$ be the coalition containing the transit and content ISPs in S that is required to serve the content q requested by the region r and the eyeball ISP in region r. Let $S_r = \cup_{q \in \mathcal{Q}_r} S_{r,q}$. Then, the coalition game for each decomposed S_r and $S_{r,q}$ is *canonical* (i.e., each player's marginal contribution is 0 or the worth of S_r and $S_{r,q}$). It can be easily shown that for a canonical coalition game the Shapley value can be simply characterized based on the notion of *Shapley portion*, as stated in the next.

Theorem 1 (Shapley value in RSG for a coalition) For any coalition $S \subset \mathcal{N}$, $i \in S$, the Shapley value $\varphi^i(S)$ is:

$$\varphi^i(S) = \sum_{r \in \mathcal{R}} \left(\phi^i(S_r) \cdot \alpha_r X_r + \sum_{q \in \mathcal{Q}_r} \phi^i(S_{r,q}) \cdot \beta_q \bar{X}_{r,q} \right),$$

where $\phi^i(M)$ is the Shapley portion of the player i in the given coalition M, defined by:

$$\phi^i(M) \triangleq \frac{1}{|M|!} \sum_{\pi \in \Pi} \mathbf{1}_{[\Delta_i(v, M(\pi,i))>0]},$$

where $\mathbf{1}_{[\cdot]}$ is the indicator function.

Clearly, for S, $i \notin S$, $\varphi^i(S) = 0$. Intuitively, the Shapley portion is the portion of orderings (out of all possible orderings made by the players in the coalition M) that has positive marginal contribution of the player i with respect to the players preceding i in each corresponding ordering. Of particular interest is the Shapley value for the grand coalition, i.e., $S = \mathcal{N}$.

B. Stability of Under-demanded Network

In this subsection, we explore the stability condition of the grand coalition under the SV based revenue sharing. Recall Definition 2 for the notion of stability, from which the grand coalition is stable iff for any coalition $S \subseteq \mathcal{N}$, there exists an ISP $i \in S$ which has the Shapley value under the grand coalition that is larger than or equal to that in the coalition S. We call such ISP in RSG *Shapley-advocating* ISP. Theorem 2 states that in under-demanded networks, the grand coalition is provably stable under the SV based revenue sharing in RSG.

Theorem 2 (Stability of GC in under-demanded network) In under-demanded networks of RSG, the grand coalition is stable under the SV based revenue sharing.

We omit the proof for brevity and summarize the sketch of the proof here. For two coalitions, the grand coalition $\mathcal{N}$ and a sub-coalition $S \subset \mathcal{N}$, we first choose a transit ISP directly connected to any eyeball ISP in S. We prove that the chosen transit ISP is a Shapley-advocating ISP. This becomes true due to the fact that transit ISPs are responsible for connecting eyeball and content ISPs, and thus as more ISPs are connected (e.g., $\mathcal{N}$ has more ISPs than S), their Shapley portion as well as the total worth tend to increase.

Note that in [3], [20], it has been *conjectured* that the grand coalition is stable under SV, and the revenue sharing based on SV is very close to the bilateral revenue exchange among ISPs. Theorem 2 states that the grand coalition is always stable under under-demanded networks. However, it is not clear about the stability of grand coalition when user demands exceed network capacity, which we will study in the next section.

C. Stability of Over-demanded Network

We now investigate the stability of the grand coalition when the network is over-demanded. Note that to guarantee the stability, it is necessary to find a Shapley-advocating ISP, for which we need to compare the Shapley values in $\mathcal{N}$ and any sub-coalition $S \subset \mathcal{N}$. The distinctive feature of the over-demanded case as compared with the under-demanded one lies in the fact that the total worth of a given coalition S depends on over-demand policies as well as configurations (e.g., players and topology) of S. To be more specific, a region that is over-demanded in $\mathcal{N}$ may *not* be over-demanded in S,

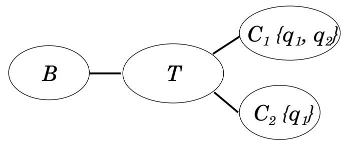

Fig. 2. Example Topology: Stability and Social Welfare

and also, the amount of traffic reduction due to the adopted over-demand policy can be significantly different in $\mathcal{N}$ and S. Only after paying due respect to these issues, we can figure out the conditions which ensure the stability of the grand coalition.

The above intuitions will be concretely exemplified by a simple network shown in Figure 2. We will show that the grand coalition can be stable or unstable, depending on the parameter values. From this example, we draw some engineering implications on the stability of the grand coalitions.

We consider the following parameters, where we omitted the region index in all notations since there is only one region:

TABLE I
PARAMETER VALUES OF THE NETWORK IN FIGURE 2

	$X, X_{q_1}, \underline{X_{q_2}}$	$\beta_{q_1}, \beta_{q_2}, \alpha$	s_{q_1}, s_{q_2}
Case 1	$100, 80, \underline{60}$	$1, 3, 1$	$3, 1$
Case 2	$100, 80, \underline{120}$	$1, 3, 1$	$3, 1$

As observed, two cases are distinguished by the values of X_{q_2} (underlined in the table), representing the user population requesting access to the content q_2. Applying Definition 2, it is easy to see that *Case 1 is stable, but Case 2 is unstable.*

Case 1 becomes stable, since for the transit ISP T, we can prove that for any coalition $S \in \mathcal{N}$, T's SV is larger in $\mathcal{N}$ than in S. However, in Case 2, for a coalition $\{B, T, C_1(q_2)\}$, all players there have larger SVs than in the grand coalition, whose reason can be explained as follows: Note that in both cases the network is over-demanded in the grand coalition, where a certain portion of the demand of user population must be reduced inevitably. In Case 1, first, the revenue per unit traffic volume for q_2 (i.e., β_{q_2}/s_{q_2}), in spite of its small user population, is relatively large (compared to q_1), which incentivizes the transit ISP to serve q_2, irrespective of q_1. Second, though the revenue per unit traffic volume for q_1 (i.e., β_{q_1}/s_{q_1}) is relatively small (compared to q_2), the user population for q_2 (i.e., X_{q_2}) is not large enough, thereby rendering the grand coalition more lucrative for the transit ISP. Thus, the transit ISP tries to contain q_1 as well in its coalition. At the end of the day, the grand coalition is preferred by the transit ISP. However, in Case 2, as the user population for the access q_2 increases and thus the network is more over-demanded in the grand coalition, the population reduction (therefore, the revenue reduction) happens, so that the coalition $\{B, T, C_1(q_2)\}$ that serves only q_2 generates larger revenues to all providers in the coalition than that in the grand coalition. To summarize, when there exists a content that pays less "fee" per unit traffic, the content is more likely to be excluded from the grand coalition as the congestion becomes severe (the aggregate demand increases).

V. Social Welfare vs. Stability

In the earlier section, we studied when the grand coalition is stable under the SV based revenue sharing. We now change our attention to under which coalition structure the total sum of the revenue distributed according to the SV is maximized. The social welfare $U(\mathcal{N}, \mathcal{P})$ in RSG of a coalition structure $\mathcal{P}$ is defined by:

$$U(\mathcal{N}, \mathcal{P}) \triangleq \sum_{S \in \mathcal{P}} v(S),$$

and our interest lies in finding the optimal coalition structure $\mathcal{P}^\star = \arg\max_{\mathcal{P}} U(\mathcal{N}, \mathcal{P})$. We again study the optimal coalition structure by dividing into two cases: under-demanded and over-demanded.

Under-demanded networks

It is easy to check that when RSG is super-additive, an optimal coalition structure is the grand coalition. Also, RSG is super-additive in under-demanded networks, since connecting more players increases the total worth without the reduction of worth due to large demands. Thus, we conclude that under-demanded networks will have the properties of the stability as well as the social welfare maximization with the grand coalition under the SV based revenue sharing.

Over-demanded networks

However, as implied in the stability, more complex scenarios can occur due to demand reduction which changes the individual as well as the total revenue. We also exemplify this using the same example in Figure 2 and Table I as that in the stability analysis for the over-demanded case. As opposed to stability, in Case 1, the social welfare is not maximized by the grand coalition, which is stable under SV. This is because in Case 2, all the players in the coalition $\{B, T, C_1(q_2)\}$ have larger revenue than in the grand coalition (thus unstable), and also in terms of the social welfare,$v(\{B, T, C_1(q_2)\})$ exceeds $v(\mathcal{N})$.

We consider another example in Figure 3 which has a different configuration from Figure 2 in terms of the contents served by the content ISPs as well as some parameter values. However, quite differently, we have that in this case the grand coalition is stable under SV and also is an optimal coalition structure. First, as for stability, Case 1 has the feature that the two contents' values are not significantly different, such that containing two contents together is beneficial to the transit ISP. In terms of the grand coalition's social-welfare maximization, as opposed to Figure 2, there is no coalition $\subset \mathcal{N}$ that has larger worth than the grand coalition.

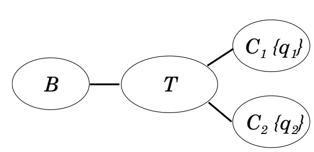

Fig. 3. Example topology 2: The grand coalition is stable and also maximizing the social welfare.

TABLE II
PARAMETER VALUES OF THE NETWORK IN FIGURE 2

	X, X_{q_1}, X_{q_2}	$\beta_{q_1}, \underline{\beta_{q_2}}, \alpha$	s_{q_1}, s_{q_2}
Case 1	$100, 75, 50$	$1, \underline{1}, 1$	$2, 1$
Case 2	$100, 75, 50$	$1, \underline{3}, 1$	$2, 1$

Impact of over-demand policy

We have so far considered the *traffic-proportional* over-demand policy where the user population is reduced in proportion to the traffic volume in the original demand for each content. We consider another over-demand policy, called *revenue-proportional* policy, which reduces the user population proportionally to the amount of the *revenue* from the original demand for each content. The actual user population $\tilde{X}_{r,q}$ is formally given by:

$$\tilde{X}_{r,q} = \min\left(X_{r,q}\frac{\beta_q n_r}{s_q \sum_{q\in\mathcal{Q}_r}\beta_q X_{r,q}}, X_{r,q}\right).$$

The rationale behind this policy is: we choose $\tilde{X}_{r,q}$, so that the total traffic volume due to the content q in region r is proportional to the q's revenue portion, i.e.,

$$s_q\tilde{X}_{r,q} = n_r\frac{\beta_q X_{r,q}}{\sum_{q\in\mathcal{Q}_r}\beta_q X_{r,q}}.$$

To examine the impact of over-demand policies, we applied the revenue-proportional policy to the example in Figure 2 where stability and social welfare's optimality do not coincide in the traffic-proportional policy based on the parameter values in Table I. We can observe that under the revenue-proportional policy, stability is guaranteed in both Cases 1 and 2, and somewhat surprisingly the social welfare is maximized also in both cases! The formal study on this new policy should be interesting and is left for the future work.

VI. CONCLUDING REMARKS AND FUTURE WORK

Inspired by recent research efforts that have shown the *rosy* prospects of (by far the most acclaimed) Shapley value based revenue sharing schemes among the Internet service providers, in this paper, we have furthered those research efforts by extending previous models to a more general network model which reflects traffic congestion, i.e., a network over-demanded by end users. By giving selected examples, we have addressed a few questions pertaining to the revenue sharing issue: the stability of over-demanded networks, the optimality of the grand coalition (in the sense of the social welfare maximization), and lastly the impact of over-demand policy.

Putting them together, we find that the prospects of Shapley value based revenue sharing is rather limited as selfish providers in over-demanded networks would break the grand coalition to form a more lucrative coalition. On top of that, the social welfare is not necessarily maximized in the grand coalition. The situation is compounded by the fact that over-demand policy may incur a drastic change to the revenue sharing scheme. Lastly, our preliminary results are intended only to arouse interests in this topic and to emphasize the necessity of realistic network models, as compared with the simpler ones espoused in previous work.

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