On the Pricing of Femtocell Services

SeYoung Yun, Yung Yi, Dongho Cho
Electrical Eng., KAIST, South Korea
syyun@comis.kaist.ac.kr
{yiyung,dhcho}@ee.kaist.ac.kr

Jeonghoon Mo
Information and Industrial Eng., Yonsei University, South Korea
j.mo@yonsei.ac.kr

ABSTRACT

The femtocell is an enabling technology to handle exponentially increasing wireless data traffic. In this paper, we study economic aspects of femtocell deployment with a game theoretic model between a provider and users. The provider provides differentiated service of macro only, or macro plus femto services and users select one, whichever maximizes their net-utilities. Femto users also decide whether to share their femto APs with other users given economic incentive. We answer the optimal pricing of providers and the optimal incentives.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication

General Terms

Management

Keywords

femtocell, pricing, game model

1. INTRODUCTION

Recently, the demand of data traffic in wireless networks has dramatically increased. Many researchers from networking and financial sectors [1, 2, 4, 5] forecast that by 2014, an average mobile user will consume 7GB of traffic per month which is 5.4 times more than today's average user consumes per month. The main drive behind this unprecedented growth is mainly due to smart mobile devices that offer ubiquitous Internet access and diverse multimedia applications. Many solutions to cope with the huge increase of mobile data exist, e.g., physical layer innovations, of which obtaining spectral efficiency via reducing cell sizes is known to provide the most powerful approach. In that context, femtocells are gaining a lot of attentions as a cheap yet viable way of achieving high spectral efficiency. Most of research on femtocell are performance-oriented, e.g., interference management through fancy power control algorithms or intelligent AP association for load balancing [3].

However, of another crucial importance is the study of the economic incentives to install femtocells from the perspective of users and the provider[1]. It may have bigger impact on the success of femtocells. Shetty and Walrand [6] studied this economics of femtocell networks and drew the research result that appropriate pricing schemes enable the increase in providers' revenue, where they assumed that a femto BS can be used only by the owner. In this paper, we extend the model, and focus on other issue of openness of femtocell services. Users have two options: (i) open his femto BS which other users can connect to, and (ii) close his femto BS for his exclusive use. The focus of this paper lies in pricing and openness of femtocell networks by studying the economic interactions between the provider and users. They are coupled via the pricing strategy by the provider and users' utility to use the service, and also the network characteristics of femtocell networks.

The main messages of this paper are summarized as follows: By allowing the open-femto services, a *significant increase* in service providers' revenue, users' utility, and the social welfare is achieved with *small subsidies* to open-femtocell users. For example, under a setting of our numerical examples, increase in the revenue of the provider ranges from 20% to about 70%, depending on the location of the generate traffic; As we have more traffic outside the femto BS that one owns, the gain becomes larger. In terms of users' utility, the increase with the open-femto service is upto 10 times at the best case than that with only closed-femto service. We also investigated the subsidy, i.e., the amount of money the provider discounts for the user who opens his femto BS. From our numerical computation, this subsidy does not have to be large, and just a small discount is enough to incentivize the users to open their femto BS.

2. SYSTEM MODEL

Consider a monopoly wireless service provider who offers mobile services to users of population N. The operator provides three different services: *mobile only*, *mobile + open femto*, and *mobile + closed femto* with different flat subscription fees of p_m, p_o and p_c. Users of the first can use macro base station (BS) only while those of second and third can use both macro BS and femto BS networks. Users of the open femto services allow other users to access their femto networks while those of closed femto do not as Figure 1. By opening the femto networks, they pay lower monthly fees, i.e. $p_o < p_c$.

We assume that a perceived utility of a user j of type γ who

[1]We use 'operator' and 'provider' interchangeably throughout this paper.

CFI '10 June 16-18, 2010, Seoul, Korea

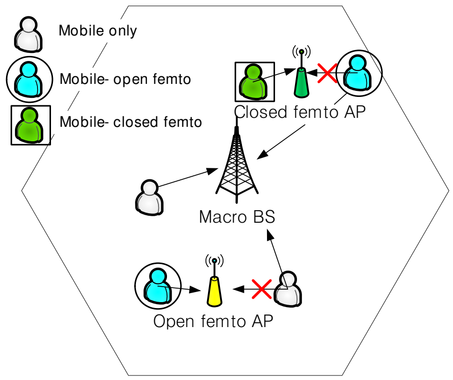


Figure 1: Links allowed for mobile only, mobile+open femto, and mobile+closed femto

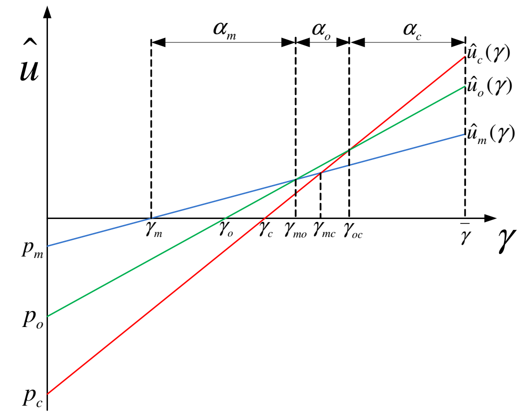


Figure 2: Net utility of users for each service type

achieves throughput x is

$$u_j(x;\gamma) = \gamma g_j(x),$$

where g_j is a nondecreasing concave function and γ is uniformly distributed between $[0, \bar{\gamma}]$ for $j \in \{m, o, c\}$. A user selects one of the three services in $\{m, o, c\}$ or decides not to select any. Then the net-utility $\hat{u}$ of a type γ user for service j is

$$\hat{u}_j(x;\gamma) = \gamma g_j(x) - p_j, \; j \in \{m, o, c\},$$

Here, p_o and p_c are the prices of femtocell service for an open and a closed AP, respectively. A user selects a service to maximize his own net-utility when his net-utility is positive or does not select any, otherwise.

Users generate packets with rate λ_i when they are inside or λ_o when they are outside. We assume that packet streams form Poisson processes. The service rate of a macro BS and that of a femto BS are exponentially distributed with rates μ_m and μ_f, respectively. We further assume that a user is inside with probability δ_i or outside with probability $\delta_o := 1 - \delta_i$, independent of anything else. From our traffic models, it seems to reasonable to set the parameter x of $g_j(x)$ with the *expected throughput*. In this paper, we assume that utility function $g_j(x) = x$ for all users. We will elaborate the computation of the expected throughput in Section 3.2.

3. STACKELBERG GAME MODEL

3.1 Two stage Game

We assume that both operator and users are selfish and try to maximize individual (expected) utility. In the first stage, the operator decides price vector $\boldsymbol{p} = (p_j : j \in \{m, o, c\})$ to maximize his revenue R by solving the following problem:

$$\textbf{Provider}: \quad \begin{array}{ll} \max_{p_m, p_o, p_c} & R = N(p_m\alpha_m + p_o\alpha_o + p_c\alpha_c) \\ s.t & p_m, p_o, p_c \geq 0, \end{array} \tag{1}$$

where α_j is a fraction of service type j users. In the second stage, a user of type γ selects a service $j^*(\gamma)$ that maximizes his net-utility or

$$\textbf{User}: \quad j^*(\gamma) = \arg \max_{j \in \{m,o,c\}} \hat{u}_j(x;\gamma), \tag{2}$$

when his maximum net-utility is positive. Otherwise, he does not select any service. As shown in Figure 2, $\boldsymbol{\alpha} = (\alpha_j : j \in \{m, o, c\})$ is a function of price levels $\boldsymbol{p}$ and data rate x. The net-utility of user type γ is positive if $\gamma > \gamma_j$ for $j \in \{m, o, c\}$. Let γ_{ij} be a point such that $\hat{u}_i(x;\gamma_{ij}) = \hat{u}_j(x;\gamma_{ij})$ for all $i, j \in \{m, o, c\}$. Then, we can find $\boldsymbol{\alpha}$ as follows:

$$\begin{aligned} \alpha_m &= (\gamma_{mo} - \gamma_m)/\bar{\gamma}, \\ \alpha_o &= (\gamma_{oc} - \gamma_{mo})/\bar{\gamma}, \\ \alpha_c &= (\bar{\gamma} - \gamma_{oc})/\bar{\gamma}. \end{aligned}$$

Users choose service m if their type $\gamma \in [\gamma_m, \gamma_{mo})$. Similarly, they choose service o or c if $\gamma \in [\gamma_{mo}, \gamma_{oc})$ or $\gamma \in [\gamma_{oc}, \bar{\gamma}]$, respectively.

3.2 Expected Throughput of Users

The average throughput of users can be obtained from the arrival and service rates as we assumed Poisson arrivals of packets and exponential service times using M/M/1 formula. The throughput might be different as the access point type : macro BS, open-femto AP and closed-femto AP, Let us define four random variable T_m, T_o^{in}, T_o^{out}, and T_c that corresponds to throughputs of macro BS, of open-femto AP given that owner is in, of open-femto AP given that owner is out, and of closed-femto AP. The expected throughput $E[T_j^s]$ of service j given status $s \in \{in, out\}$ can be computed by:

$$\mathbb{E}[T_j^s] = -\frac{1-\rho_j^s}{\rho_j^s}\log(1-\rho_j^s), \tag{3}$$

where ρ_j^s is the utilization of access point j given status s. For example, $\rho_m = \frac{\lambda_m}{\mu_m}$, $\rho_o^{in} = \frac{\lambda_o^{in}}{\mu_f}$, $\rho_o^{out} = \frac{\lambda_o^{out}}{\mu_f}$ and $\rho_c = \frac{\lambda_c}{\mu_f}$.

The total arrival rate λ_m to the macro BS is given by:

$$\lambda_m = \alpha_m N(\delta_i\lambda_i + \delta_o\lambda_o) + (\alpha_o + \alpha_c)N(1 - q_o)\delta_o\lambda_o, \tag{4}$$

where α_j is the fraction of service j users for $j \in \{m, o, c\}$ and q_o is the probability that a user is served by an open femto AP when he is outside. In a similar way, we can compute total arrival rates to femto APs as follows:

$$\lambda_o^{in} = \lambda_i + (\alpha_o + \alpha_c)q_o\delta_o\lambda_o/\alpha_o, \tag{5}$$

$$\lambda_o^{out} = (\alpha_o + \alpha_c)q_o\delta_o\lambda_o/\alpha_o, \tag{6}$$

$$\lambda_c = \lambda_i. \tag{7}$$

With the expected throughputs of (3), we can compute the expected throughputs for each service, which we denote by, x_m, x_o, x_c given $\alpha = (\alpha_m, \alpha_o, \alpha_c)$. Recall that we assumed that $g(x) = x$, and thus $\gamma x_j, j \in m, o, c$ becomes the utilities of users with each service.

$$x_m = \mathbb{E}[T_m], \tag{8}$$

$$x_o = \frac{\lambda_i \delta_i E[T_o^{in}]}{\lambda_i \delta_i + \lambda_o \delta_o} + \frac{\lambda_o \delta_o \{(1-q_o)E[T_m] + q_o(\delta_i E[T_o^{in}] + \delta_o E[T_o^{out}])\}}{\lambda_i \delta_i + \lambda_o \delta_o}, \tag{9}$$

$$x_c = \frac{\lambda_i \delta_i E[T_c]}{\lambda_i \delta_i + \lambda_o \delta_o} + \frac{\lambda_o \delta_o \{(1-q_o)E[T_m] + q_o(\delta_i E[T_o^{in}] + \delta_o E[T_o^{out}])\}}{\lambda_i \delta_i + \lambda_o \delta_o}. \tag{10}$$

An intuitive explanation of the results in the above is as follows: when an open-femto user is inside, the packet generated from the user is served from his femto AP. On the other hand, when the open-femto user is outside, he can use one of other open-femto APs or the macro BS. If we let q_o be the probability that open-femto AP is available, then with probability $q_o \delta_i$, he can use a femto AP with owner inside. With probability $q_o \delta_o$, he can use a femto AP with owner outside. Similarly, the probabilities for that a closed-femto user is served by BS type M, I, O, and C are $\delta_o(1-q_o), \delta_o q_o \delta_i, \delta_o q_o \delta_o$ and δ_i. Besides, mobile-only users are always served from macro BS. In this paper, q_o is computed as

$$q_o = 1 - (1-\beta)^{\alpha_o N}, \tag{11}$$

since each open-femto AP has the same amount of coverage area which is β fraction of the coverage of the macro BS.

3.3 Equilibrium $(\boldsymbol{p}^\star, \boldsymbol{\alpha}^\star)$

Solving the two-stage game is non-trivial due to inter-coupling between $\boldsymbol{\alpha}$ and $\boldsymbol{p}$. Typically, we use backward induction to solve the sequential game, i.e, for given $\boldsymbol{p}$, we solve the user's problem to find $\boldsymbol{\alpha}$. Then, we optimize the first stage game to decide the price. However, this problem requires a Newton-type backward induction as it is a fixed point problem. Observe that even for a given $\boldsymbol{p}$, computing $\boldsymbol{\alpha}$ is non-trivial because the net utility of each user affects $\boldsymbol{\alpha}$ due to the problem **User**, and also $\boldsymbol{\alpha}$ affects the net utility due to the expected throughput's dependence of $\boldsymbol{\alpha}$. To show the dependency, we use $x_j(\boldsymbol{\alpha}), j \in \{m, o, c\}$.

The following theorem enables us to solve the problem very efficiently. Let us define $\mathcal{A}$ given by:

$$\mathcal{A} = \{\boldsymbol{\alpha} | x_m(\boldsymbol{\alpha}) \le x_o(\boldsymbol{\alpha}) \le x_c(\boldsymbol{\alpha})\}.$$

THEOREM 3.1. *For all $\boldsymbol{\alpha} \in \mathcal{A}$, the optimal revenues of the provider in (1) are the same.*

We omit the proof due to page limitation but we provide the sketch of proof. Note that for a given $\boldsymbol{\alpha}$, there may exist multiple price vectors that leads to the $\boldsymbol{\alpha}$. Recall that if a price vector is given, users selects their service to maximize their own utility (i.e., by solving **User**), which results in some $\boldsymbol{\alpha}$. Since the revenue is computed as $N(\alpha_m p_m + \alpha_o p_o + \alpha_c p_c)$, the prices for service type $j \in \{m, o, c\}$ where $\alpha_j = 0$ do not contribute to the revenue. Thus, it suffices to consider only when $p_m, p_o, p_c > 0$, in which case, if $\boldsymbol{\alpha} \in \mathcal{A}$, we can prove that the resulting revenue is uniquely determined. Denote by $\mathcal{P}(\boldsymbol{\alpha})$ the set of all $\boldsymbol{p}$ that lead to the $\boldsymbol{\alpha}$ in the way as described above.

Since, from Theorem 3.1, it suffices to use any $\boldsymbol{p}$ in the set $\mathcal{P}(\boldsymbol{\alpha})$, we will use a $\boldsymbol{p}^* \in \mathcal{P}(\boldsymbol{\alpha}^*)$, which satisfies

$$p_m^* = (1 - \alpha_m - \alpha_o - \alpha_c) g_m(x), \tag{12}$$
$$p_o^* = (1 - \alpha_o - \alpha_c)(g_o(x) - g_m(x)) + p_m^*, \tag{13}$$
$$p_c^* = (1 - \alpha_c)(g_c(x) - g_o(x)) + p_o^*, \tag{14}$$

for the optimal price. Therefore, we can have the closed form of the revenue expressed only by $\boldsymbol{\alpha}$.

4. NUMERICAL EXAMPLES

We now provide numerical examples with emphasis on revenue of the provider, social welfare, and users' payoff, where Users' payoff refers to aggregate net-utility over users and social welfare corresponds to the system-wide "happiness", referring to the sum of revenue and users' payoff.

As in [6], the network traffic density is defined as

$$c_o \;:=\; \frac{\mu_m}{(\delta_i \lambda_i + \delta_o \lambda_o) N}, \tag{15}$$

Intuitively, c_o refers to per-user traffic density from the perspective of the macro BS, when every user uses only macro BS for his service. Thus, as c_o decreases, traffic density becomes more heavy. For example, the current HSDPA networks support 3.6Mb/s. When there exists 100 possible users in the cell and they generate 36Kb/s, c_o becomes 1. The parameters used in the numerical example is described in Table 1.

Table 1: Parameter values in numerical examples

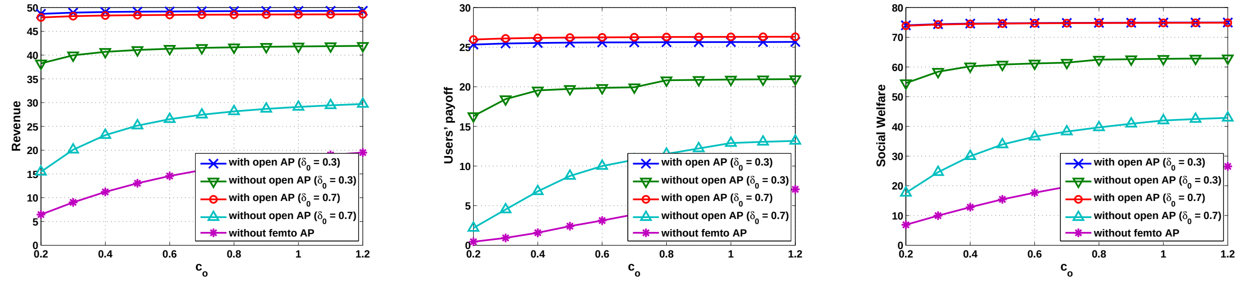
Parameter	Description	Value
N	Number of users	100
λ_i/λ_o	Traffic density ratio	1
μ_f/μ_m	Process time ratio	2
$\bar{\gamma}$	Maximum user type	1
β	Open femto coverage factor	0.05
δ_o	Fraction outside	0.3

Figures 3 (a), (b) and (c) show the users' payoff, the revenue of the provider, and the social welfare, respectively. They show that with femtocell service, those three measure increase overall in our parameter setting. Moreover, introducing open-femto service is advantageous to both users and providers.

For example, when $c_o = 1$, we obtain about 20% of revenue increase when 70% of traffic is generated from inside. When the portion of the outside-traffic increases, the gain due to open-femto APs increases, since the open APs off-load outside traffic more. For example, when 70% of traffic is generated from outside, the gain amounts to 67%.

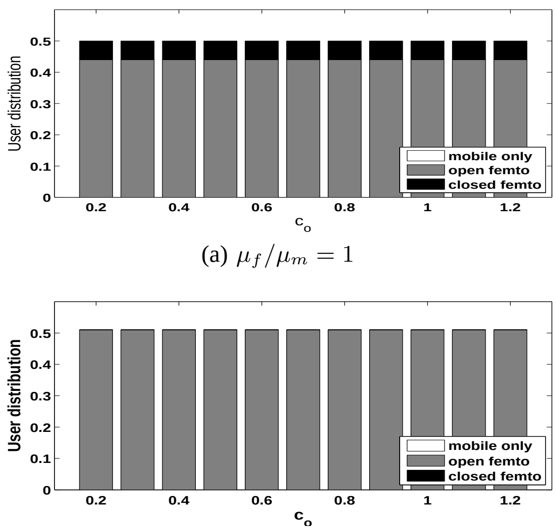
Note that by introducing open-femto APs, both the provider and users become much happier. We observe that users' payoff with open femto APs is up to 11 times more than that with only closed femto AP. Besides, the gain of open-femto APs becomes larger as network traffic density increases, which comes from off-loading traffic from the macro cell to open-femto APs. For example, when 70% of traffic is from outside and network traffic density is 0.2, the revenue of the operator with open-femto APs is 3 times more than that with only closed femto AP.

Figure 4s (a) and (b) show the portion of the users who subscribe *mobile only*, *mobile+open femto* or *mobile+closed femto* for the two cases $\mu_f/\mu_m = 1, 2$, i.e., when the femto BS's service rate is equal to or two times more than the macro BS. In both cases, we observe that no users are subscribing to *mobile-only* service. In particular, when femto APs's service rate is faster than the macro AP,



(a) (b) (c)

Figure 3: (a) Revenue, (b) Users' payoff and (c) Social welfare vs c_o



(a) $\mu_f/\mu_m = 1$

(b) $\mu_f/\mu_m = 2$

Figure 4: Users' service type vs c_o

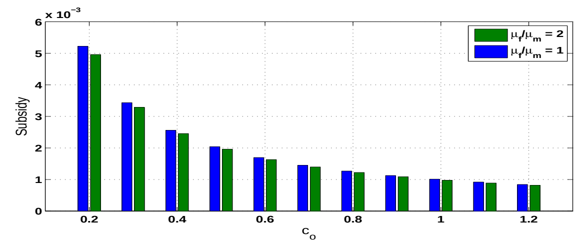


Figure 5: Subsidy for mobile+open femto vs c_o

which is a typical case, all of users are expected to subscribe to *mobile + open femto* service. Subsidy for *mobile+open femto* users, i.e., $p_c - p_o$ may not need to be large, as shown in Figure 5. Thus, with very small subsidy, we can guide users towards *mobile+open femto* service, for which users, the provider, and the social welfare significantly increase.

5. CONCLUDING REMARKS

This paper provides an analytical framework to study the pricing of femtocell services, and its economic impacts on users, providers, and social welfare. Our model has the following limitations, which should be relaxed for future research. First, we only considered a single cell for simplicity and tractability. Many issues due to the multi-cell environments, e.g., inter-cell interference and handoff may have further impact. Second, we did not consider to cost of femto-cell services. We expect that the cost of installing and managing femtocell networks is relatively low, compared to the cost incurred by macro cells, but it is necessary to include it in the model for more practicability. Third, our model assumed that the services are not differentiated between the femto-cell owner and the guests. In many cases, even if one is willing to open his femto cell to others, he may want to be served with some priority, which is also an interesting research topic.

Acknowledgment

This work was supported in part by KRCF and the KOSEF grant funded by the Korea government(2009-0075757).

6. REFERENCES

[1] Cisco visual networking index: Global mobile data traffic forecast update, 2009-2014, February 2010. `http://www.cisco.com/en/US/solutions/collateral/ns341/ns525/ns537/ns705/ns827/white_paper_c11-520862.html`.

[2] Data, data everywhere, February 2010. `http://www.economist.com/specialreports/displayStory.cfm?story_id=15557443`.

[3] V. Chandrasekhar, J. G. Andrews, and A. Gatherer. Femtocell networks: A survey. *IEEE Communications Magazine*, 46(9):59–67, September 2008.

[4] T. Kaneshige. AT&T iPhone users irate at idea of usage-based pricing, December 2009. `http://www.pcworld.com/article/184589/atandt_iphone_users_irate_at_idea_of_usagebased_pricing.html`.

[5] M. Reardon. Cisco predicts wireless-data explosion, Feburay 2010. `http://news.cnet.com/8301-30686_3-10449758-266.html`.

[6] N. Shetty, S. Parekh, and J. Walrand. Economics of femtocells. In *Proceedings of IEEE Globecom*, 2009.